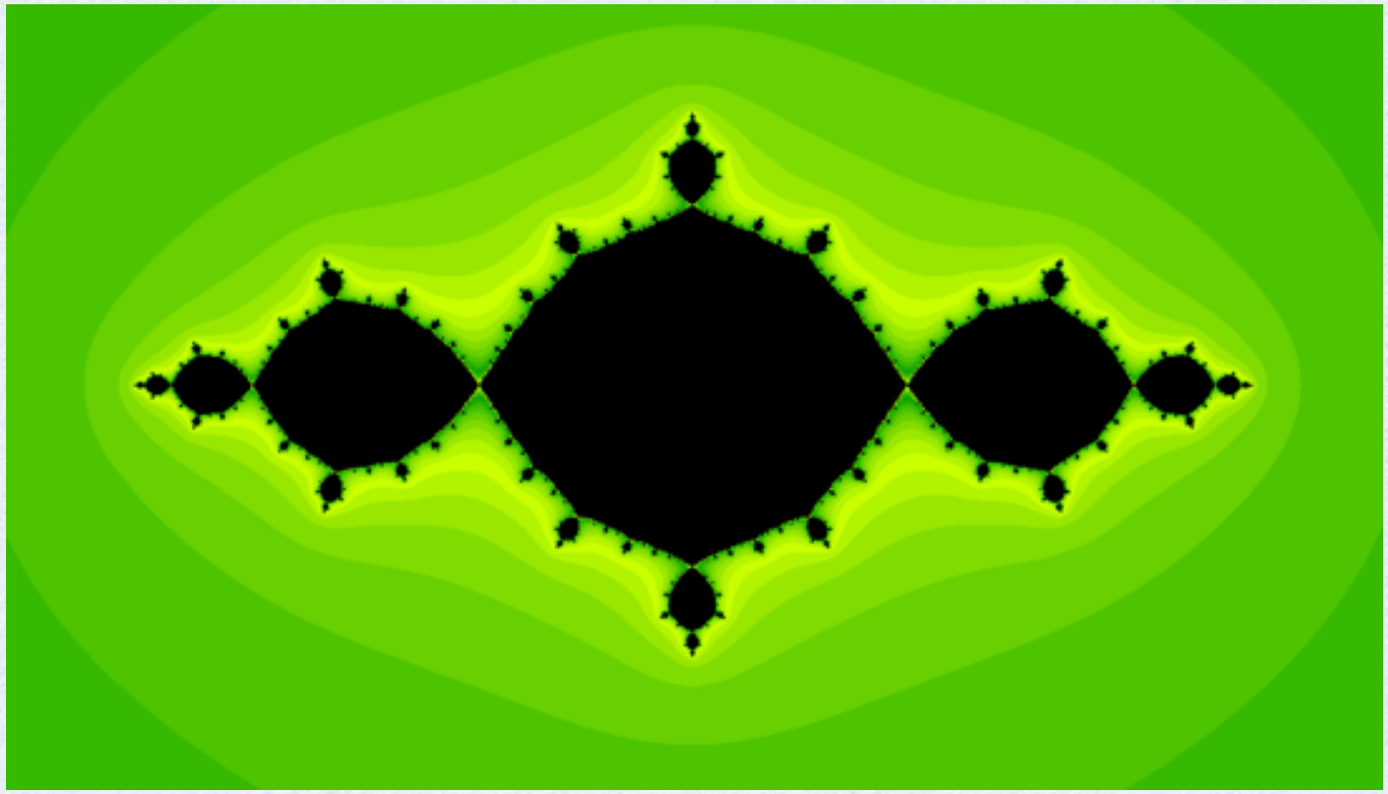
Pasting polynomials together

Sarah C. Koch

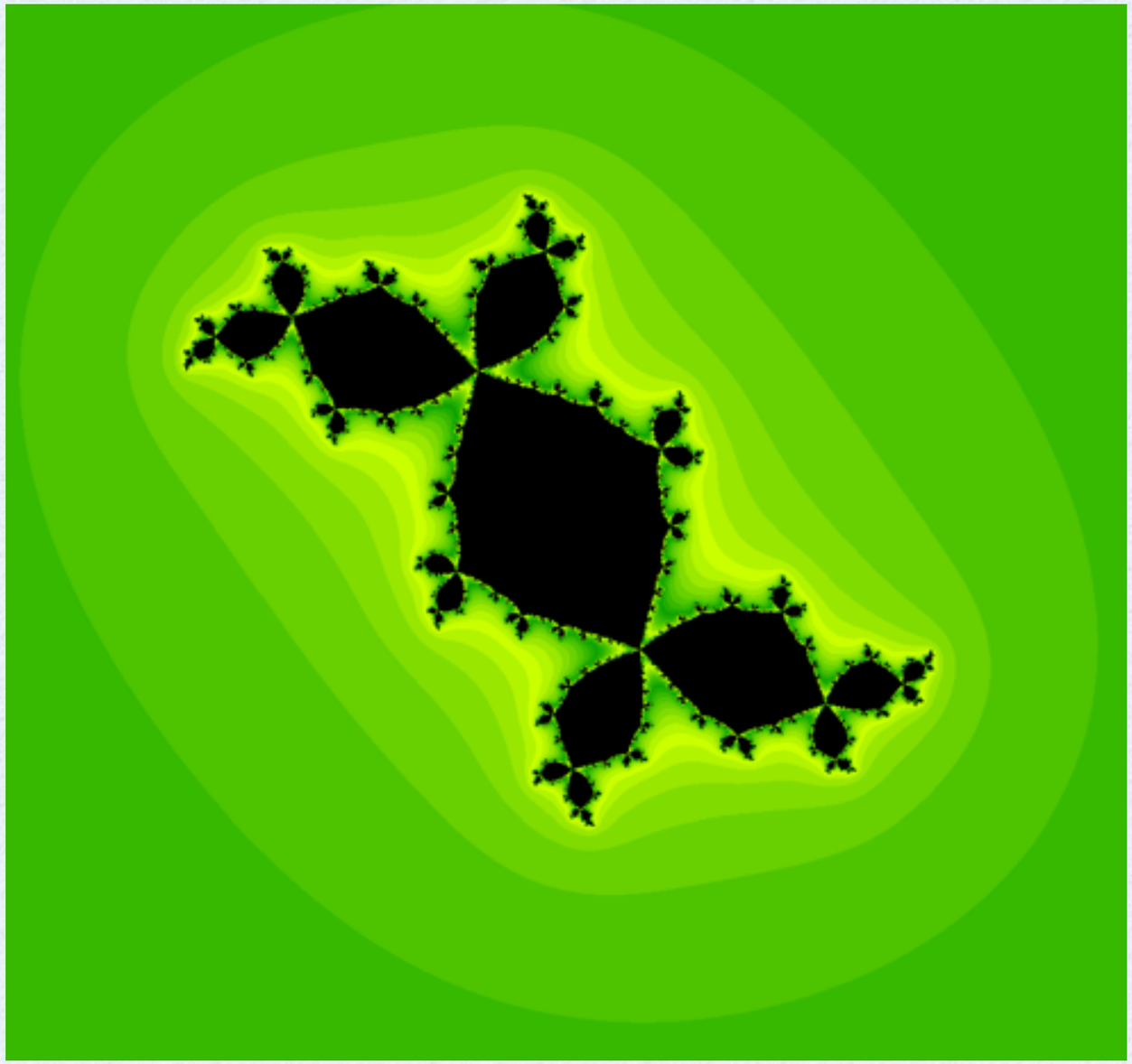
University of Michigan

The Basilica



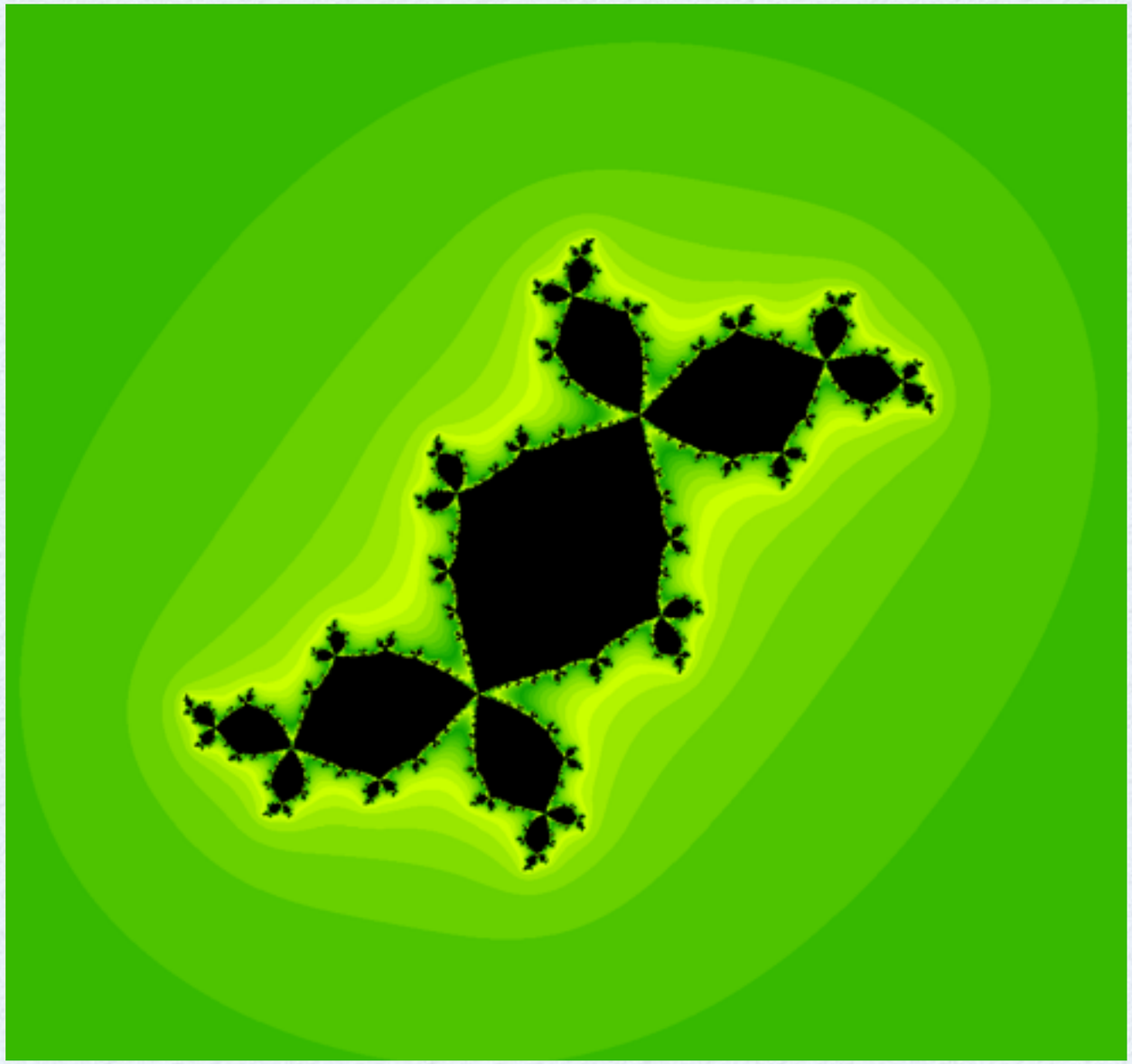
San Marco Cathedral
Venice, Italy

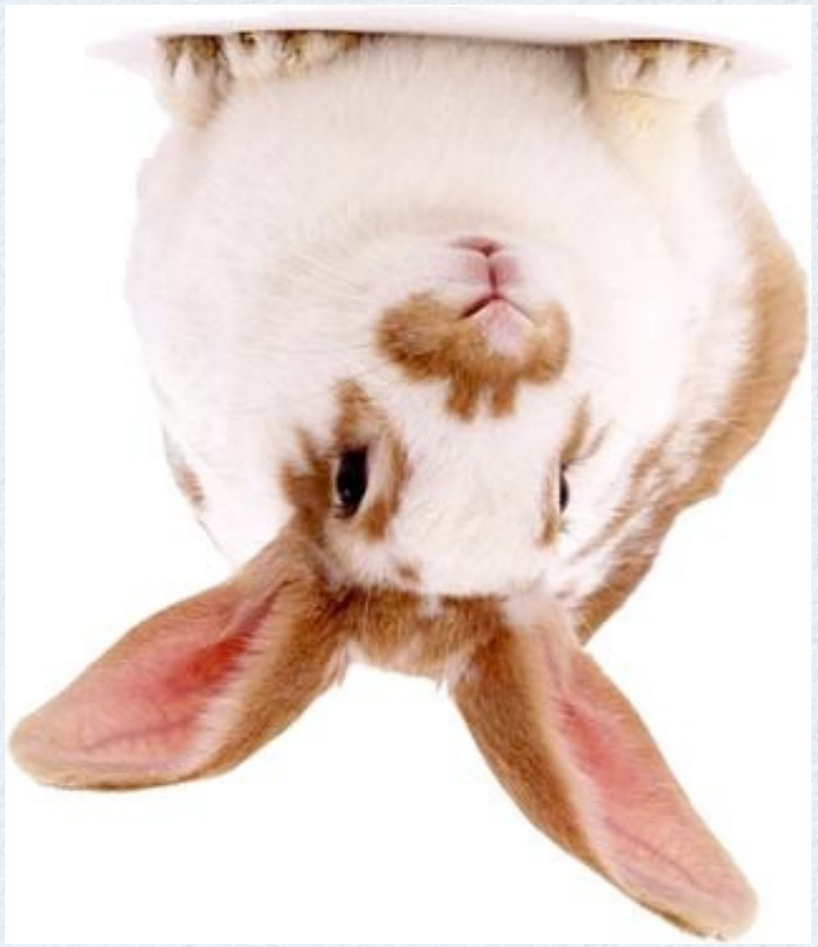
The Rabbit



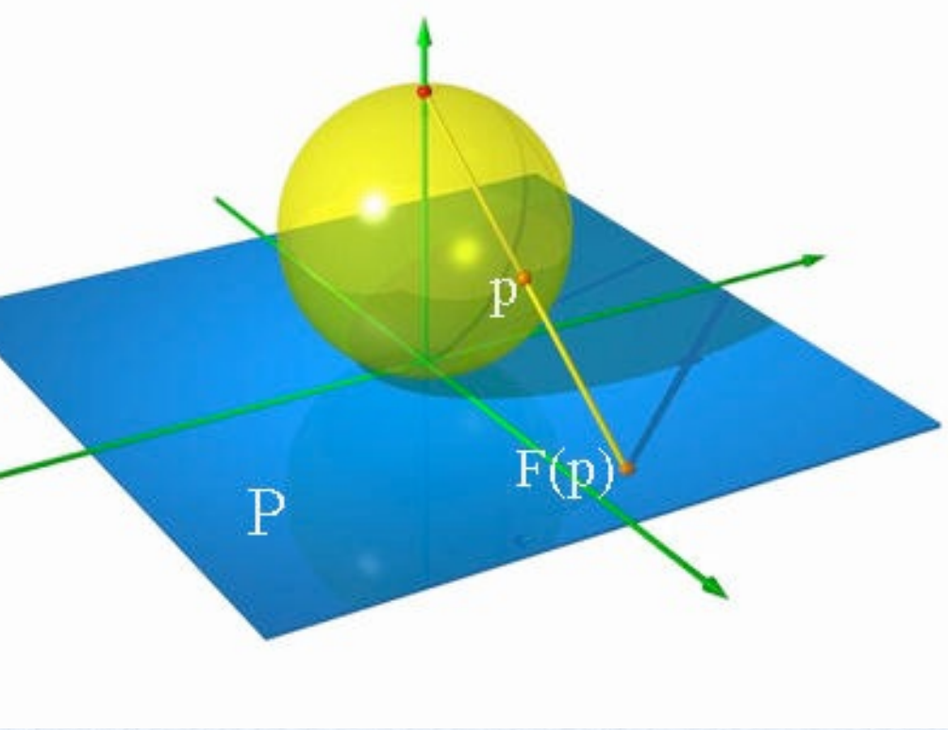
$$p(z) = z^2 + (-0.1226 + 0.7449i)$$

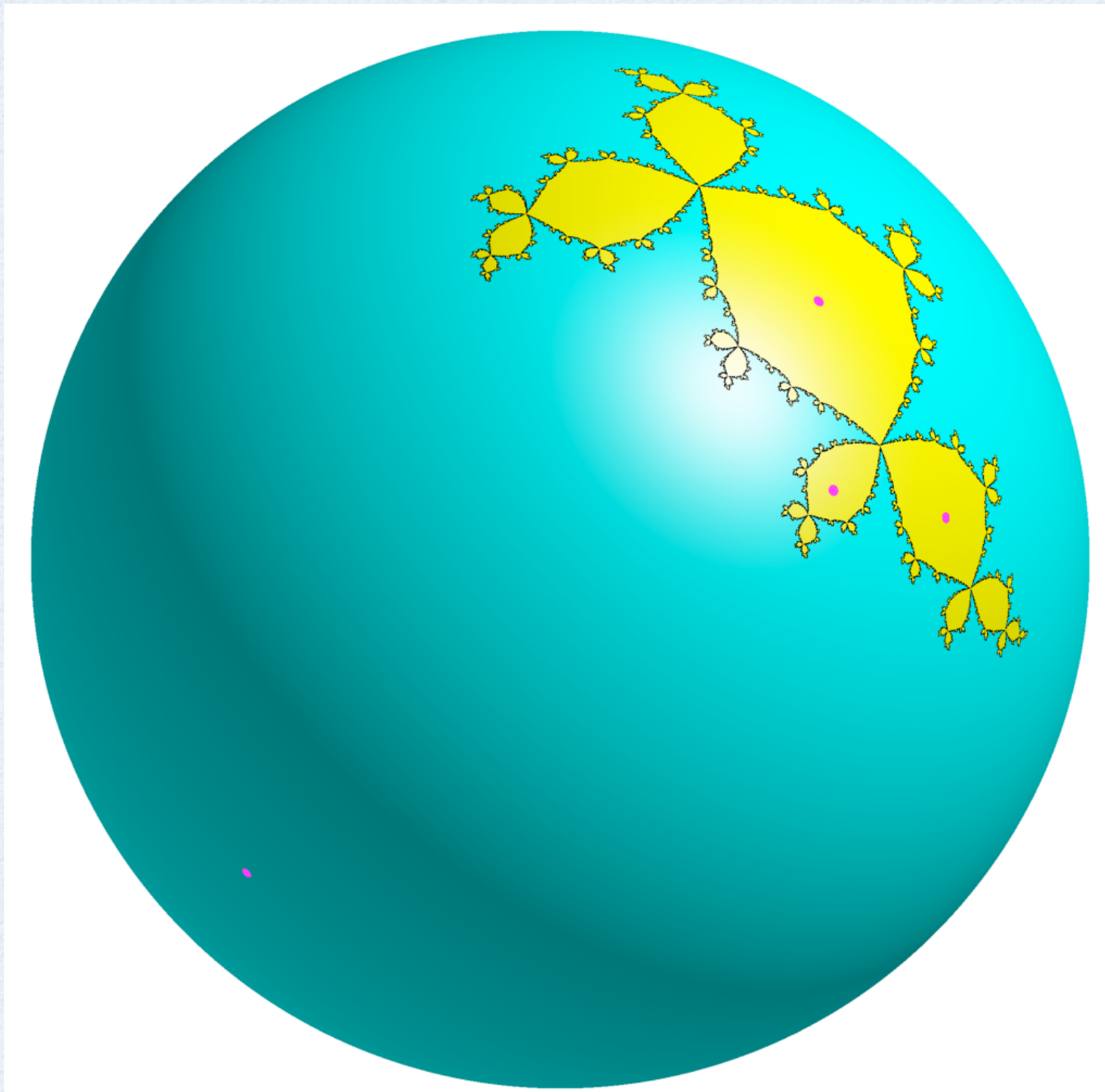
The Corabbit





$$p(z) = z^2 + (-0.1226 - 0.7449i)$$

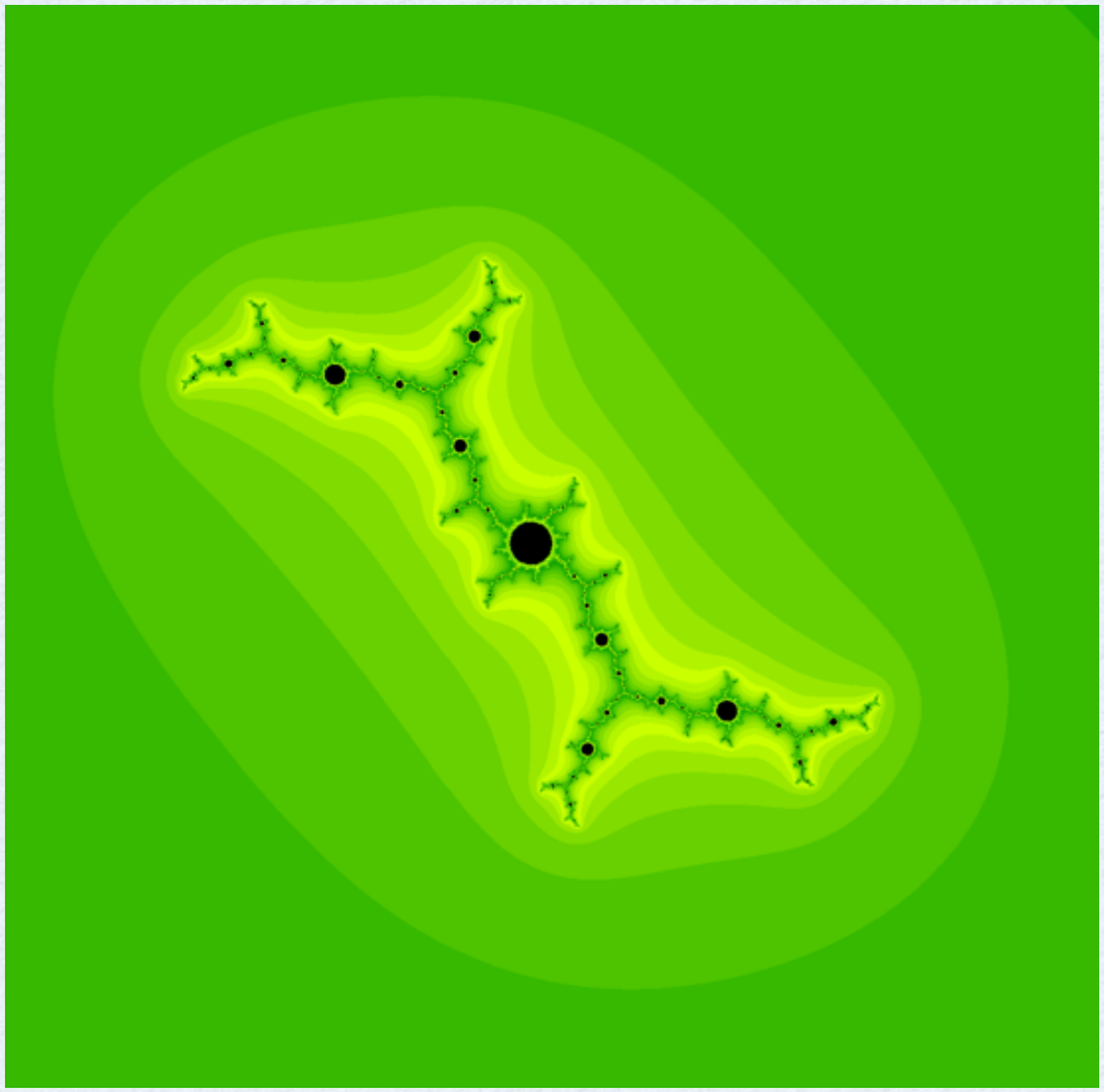

p
F(p)
P



A dendrite

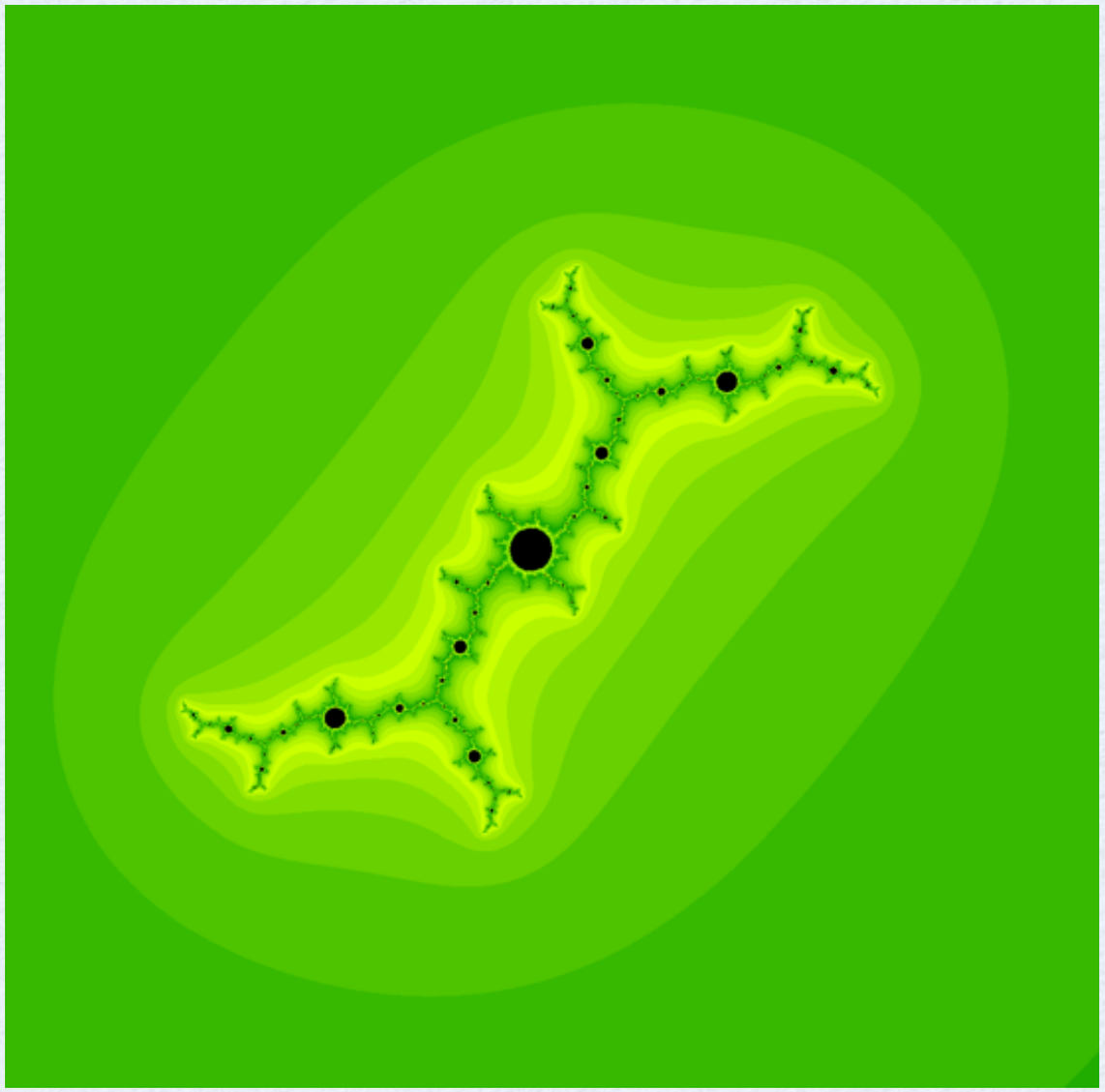
$$p(z) = z^2 + i$$

Kokopelli

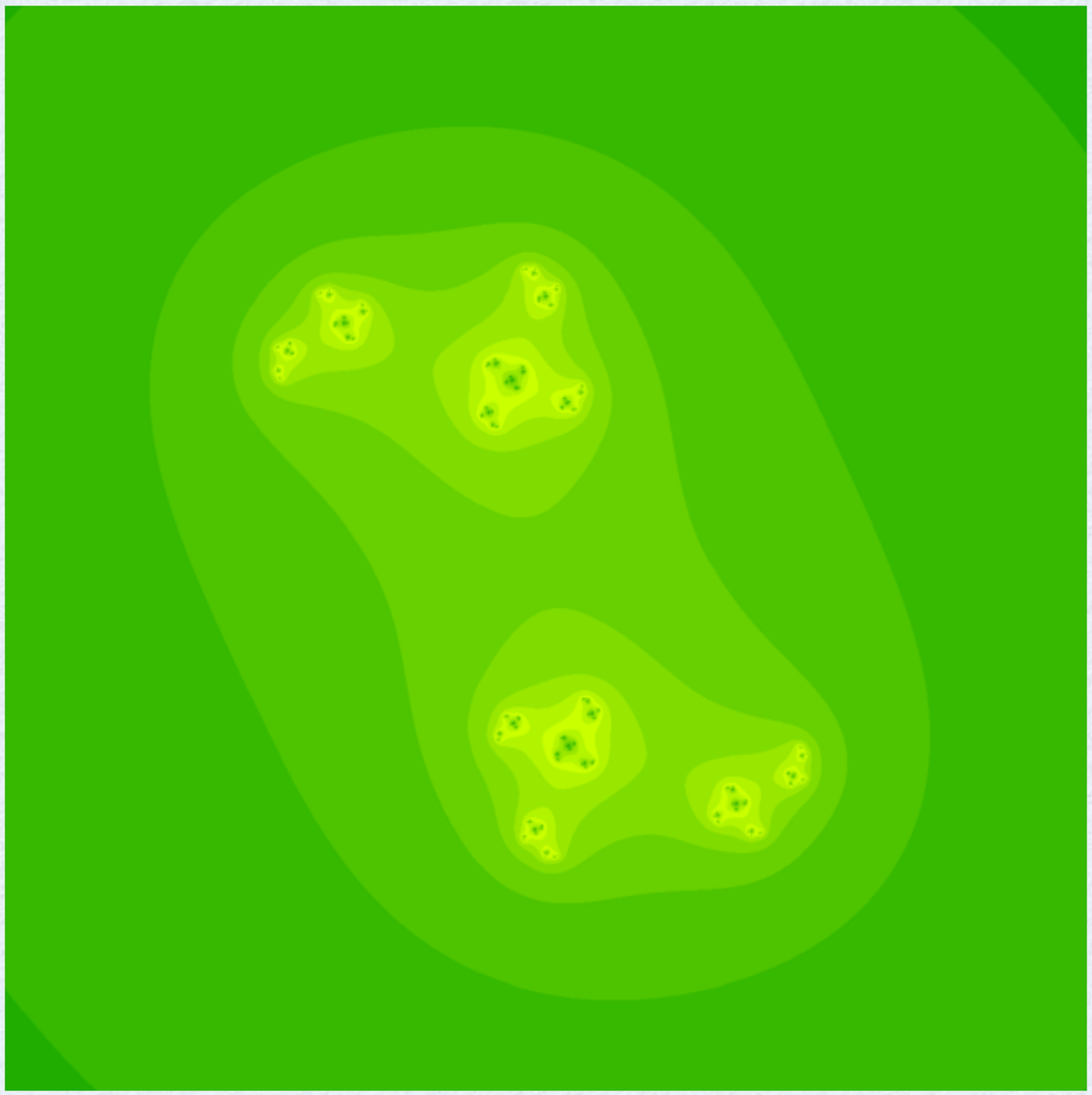


$$p(z) = z^2 - 0.156 + 1.302i$$

Cokokopelli

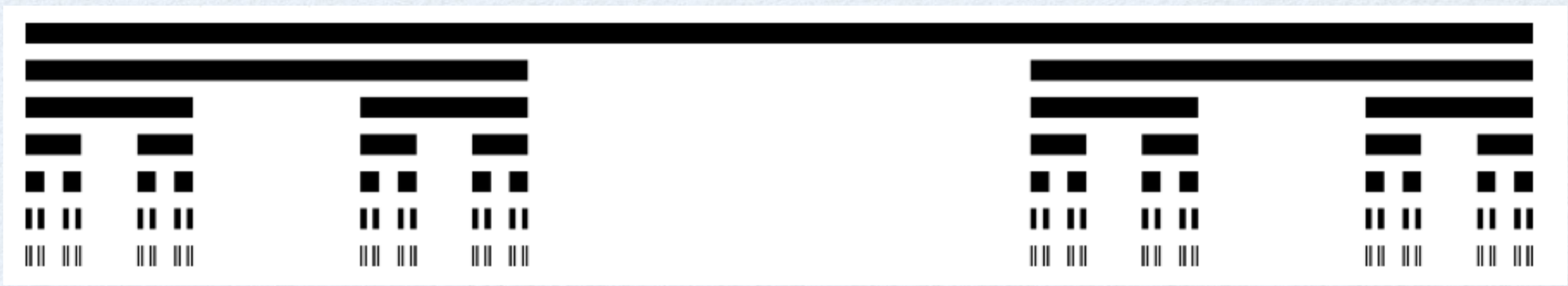


$$p(z) = z^2 - 0.156 - 1.302i$$



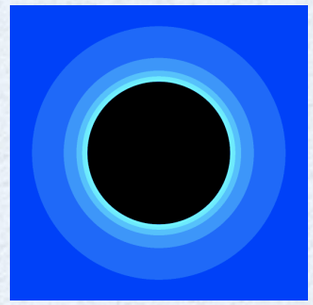
$$p(z) = z^2 + (0.5 + i)$$

A Cantor set

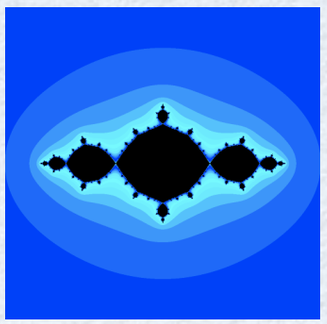


Keeping track of shapes

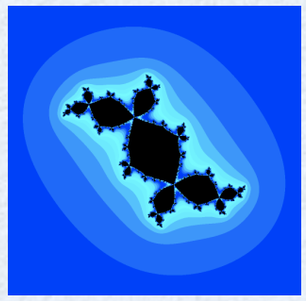
z^2



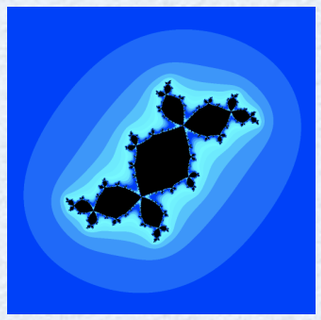
$z^2 - 1$ basilica



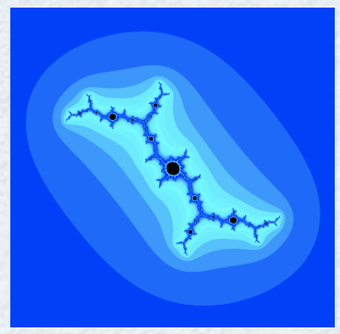
$z^2 + (-0.1 + 0.75i)$ rabbit



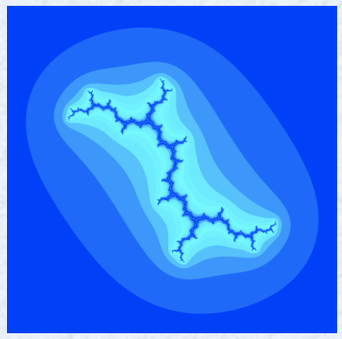
$z^2 + \overline{(-0.1 + 0.75i)}$ corabbit

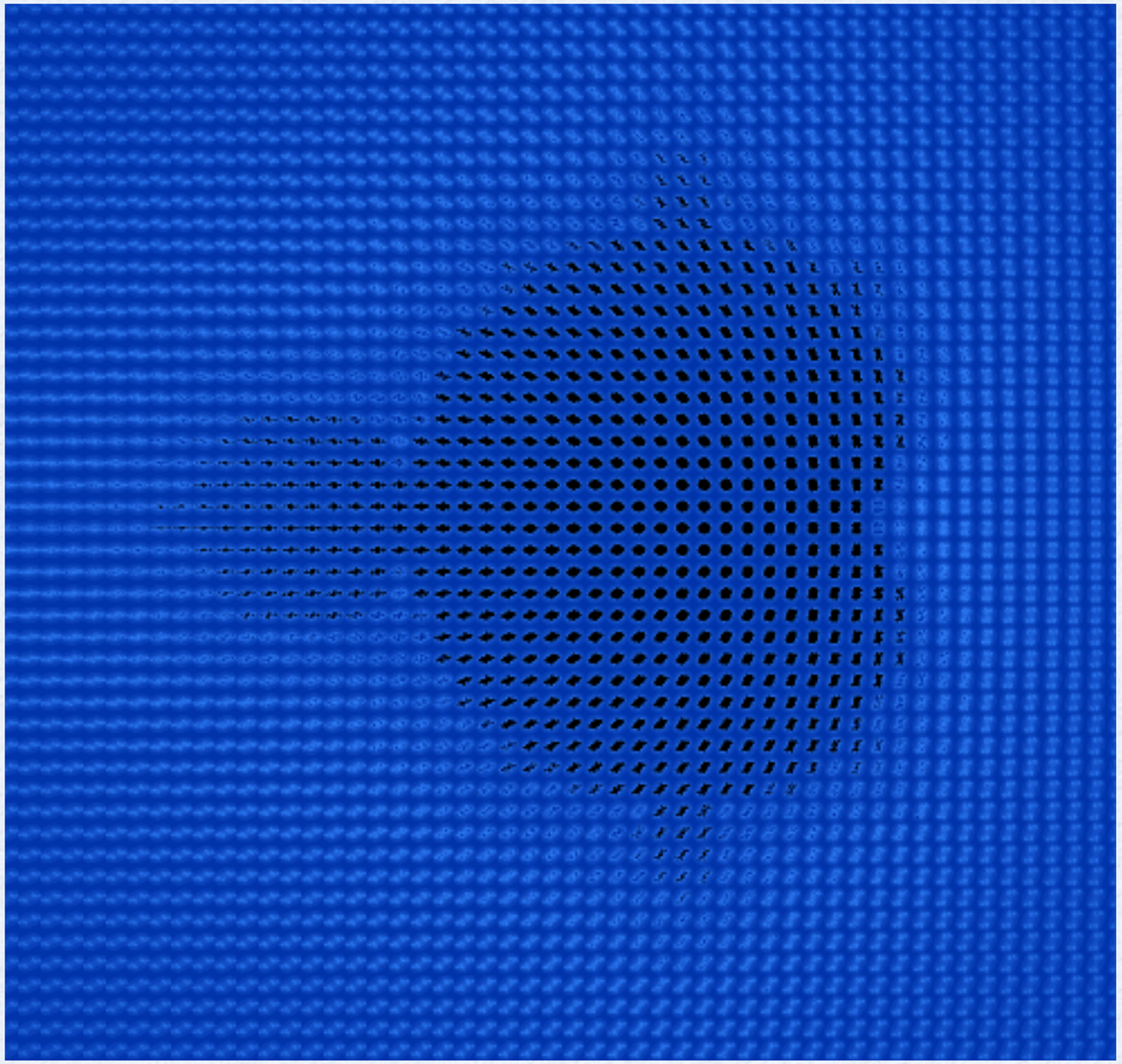


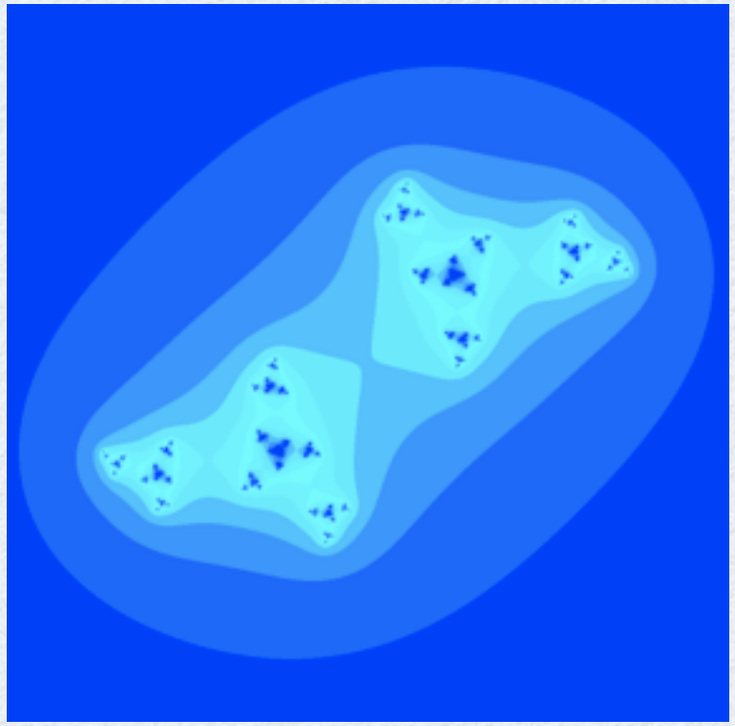
$z^2 - 0.156 + 1.302i$ kokopelli

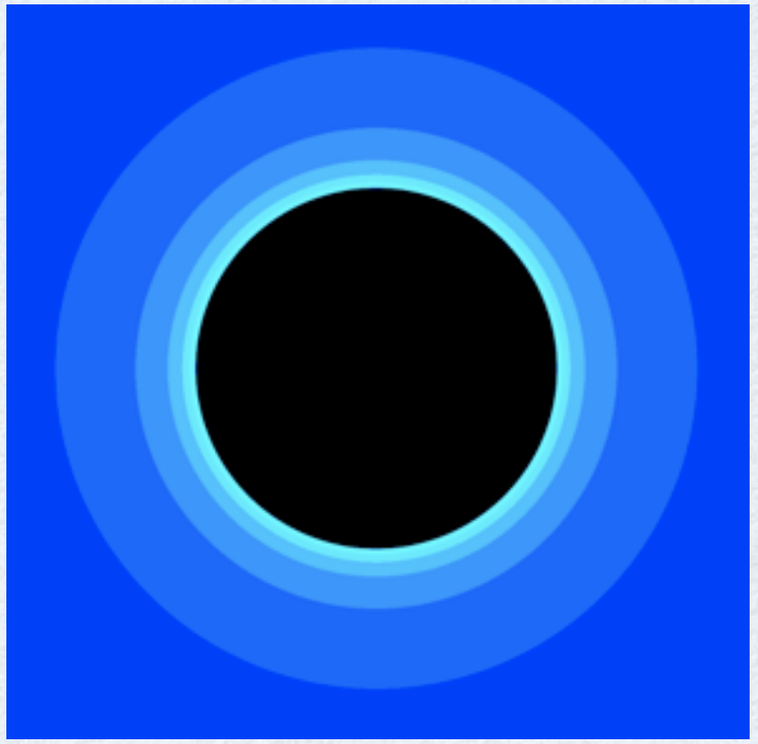


$z^2 + i$ dendrite

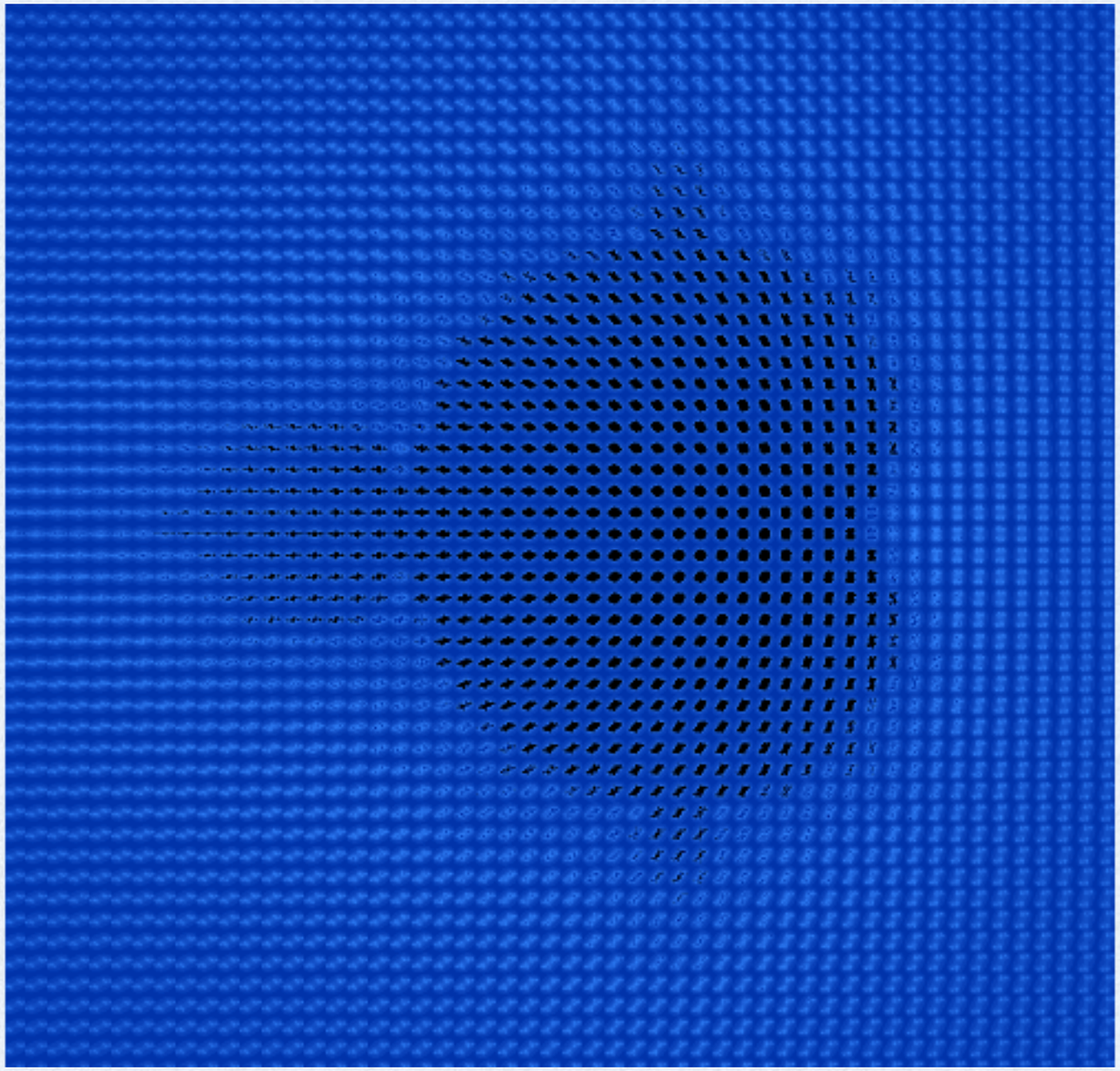




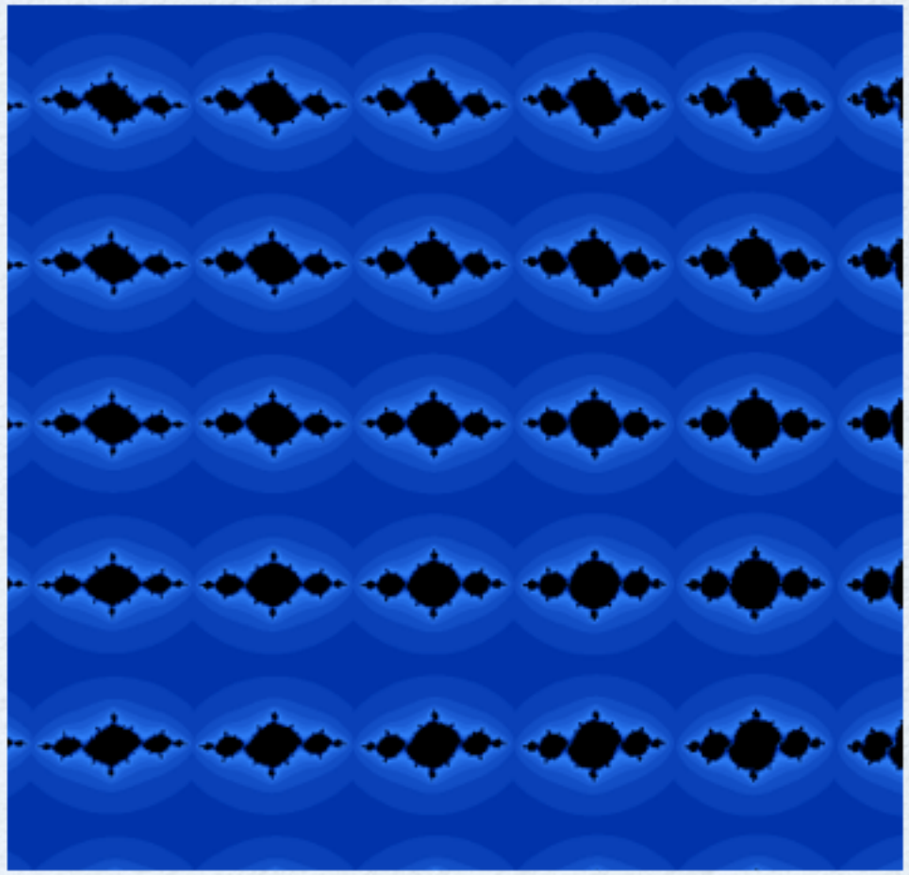




Parameter space: coloring scheme?



c plane



z^2

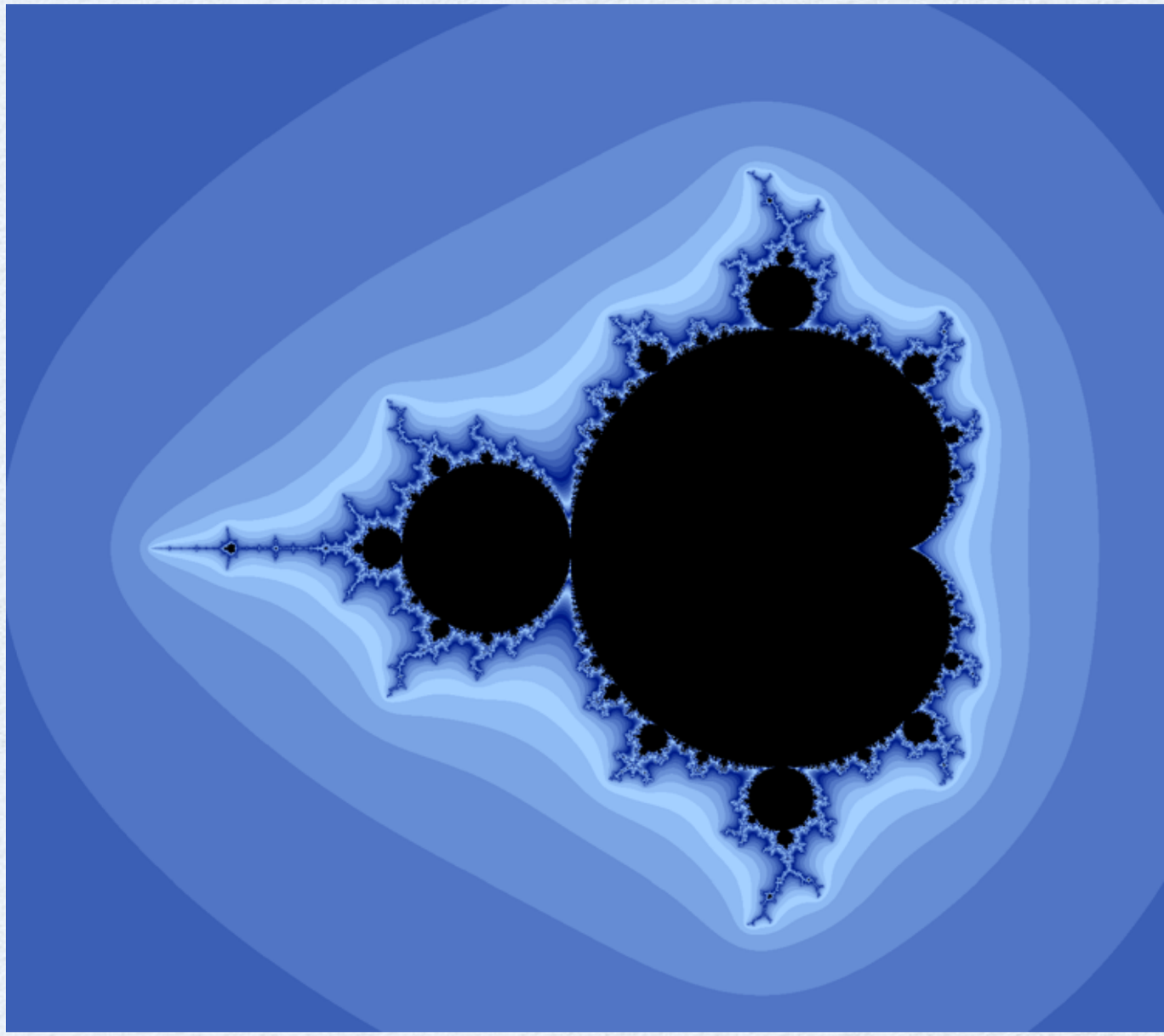
$z^2 - 1$ basilica

$z^2 + (-0.1 + 0.75i)$ rabbit

$z^2 + \overline{(-0.1 + 0.75i)}$ corabbit

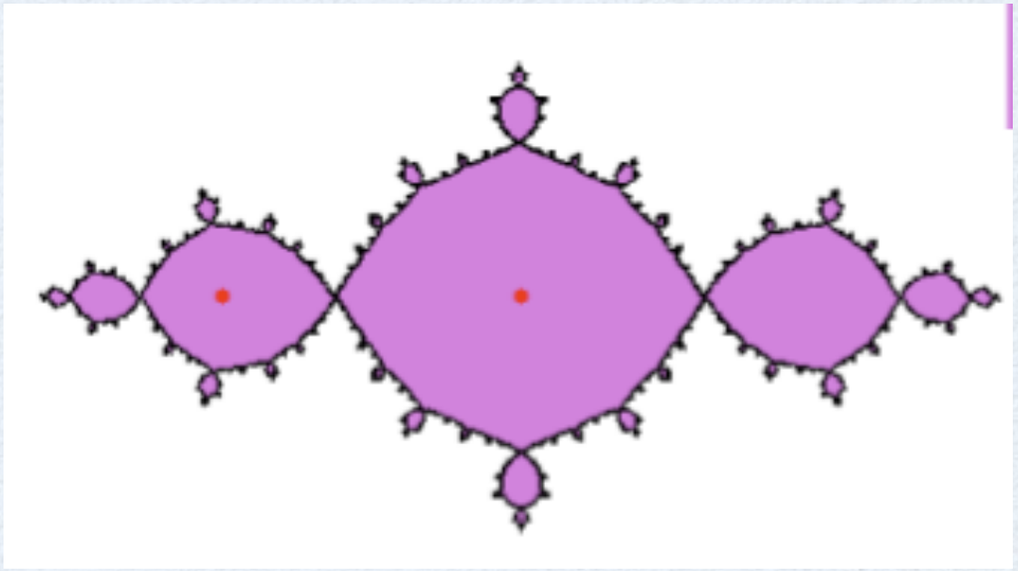
$z^2 - 0.156 + 1.302i$ kokopelli

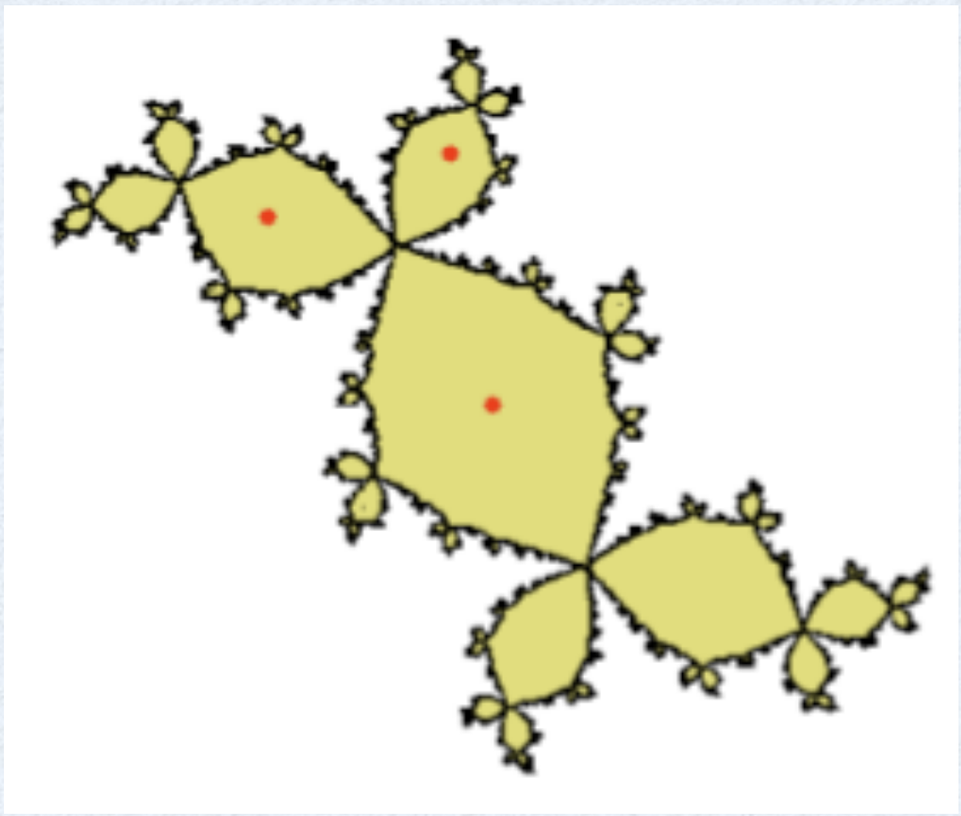
$z^2 + i$ dendrite



The Mandelbrot Set

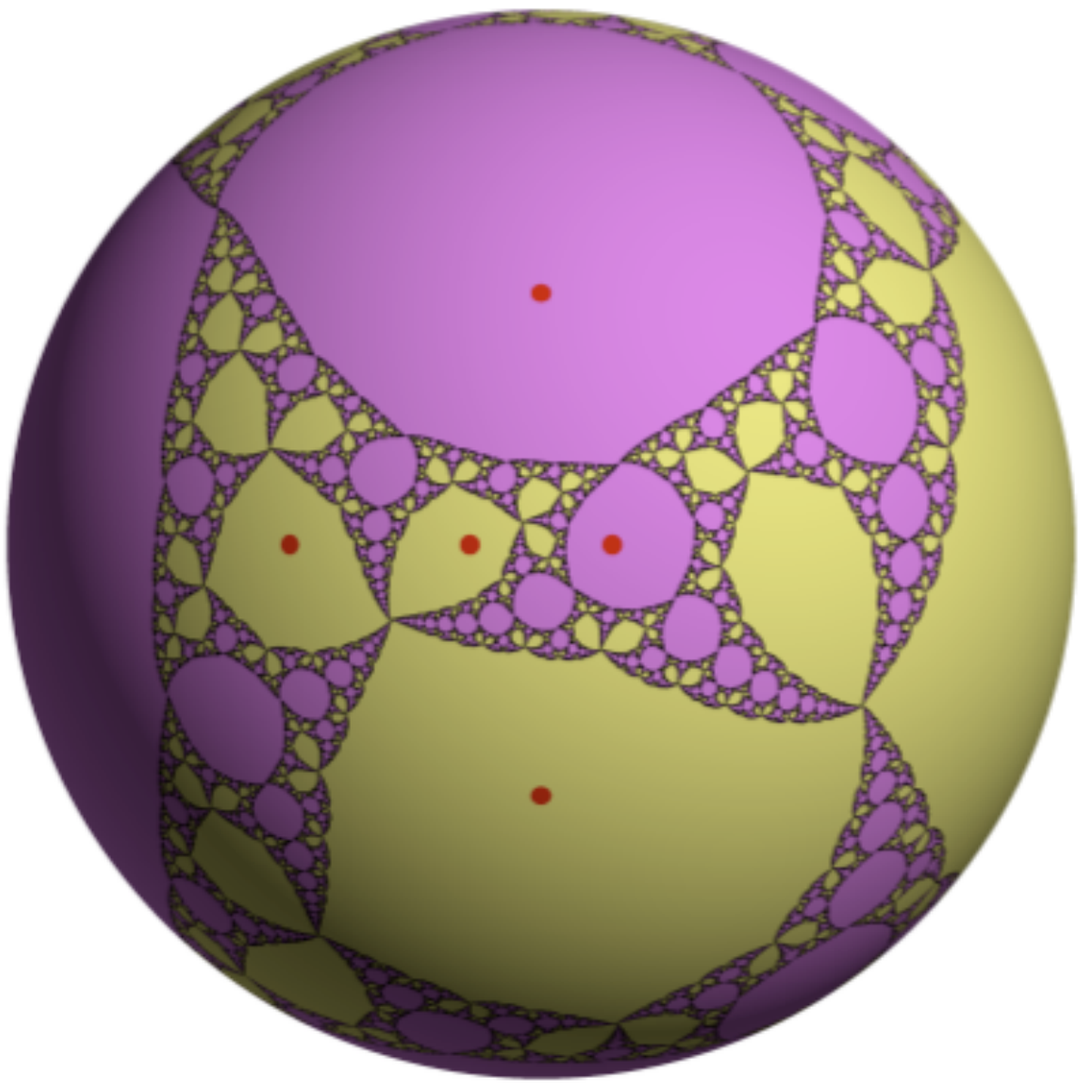
the basilica



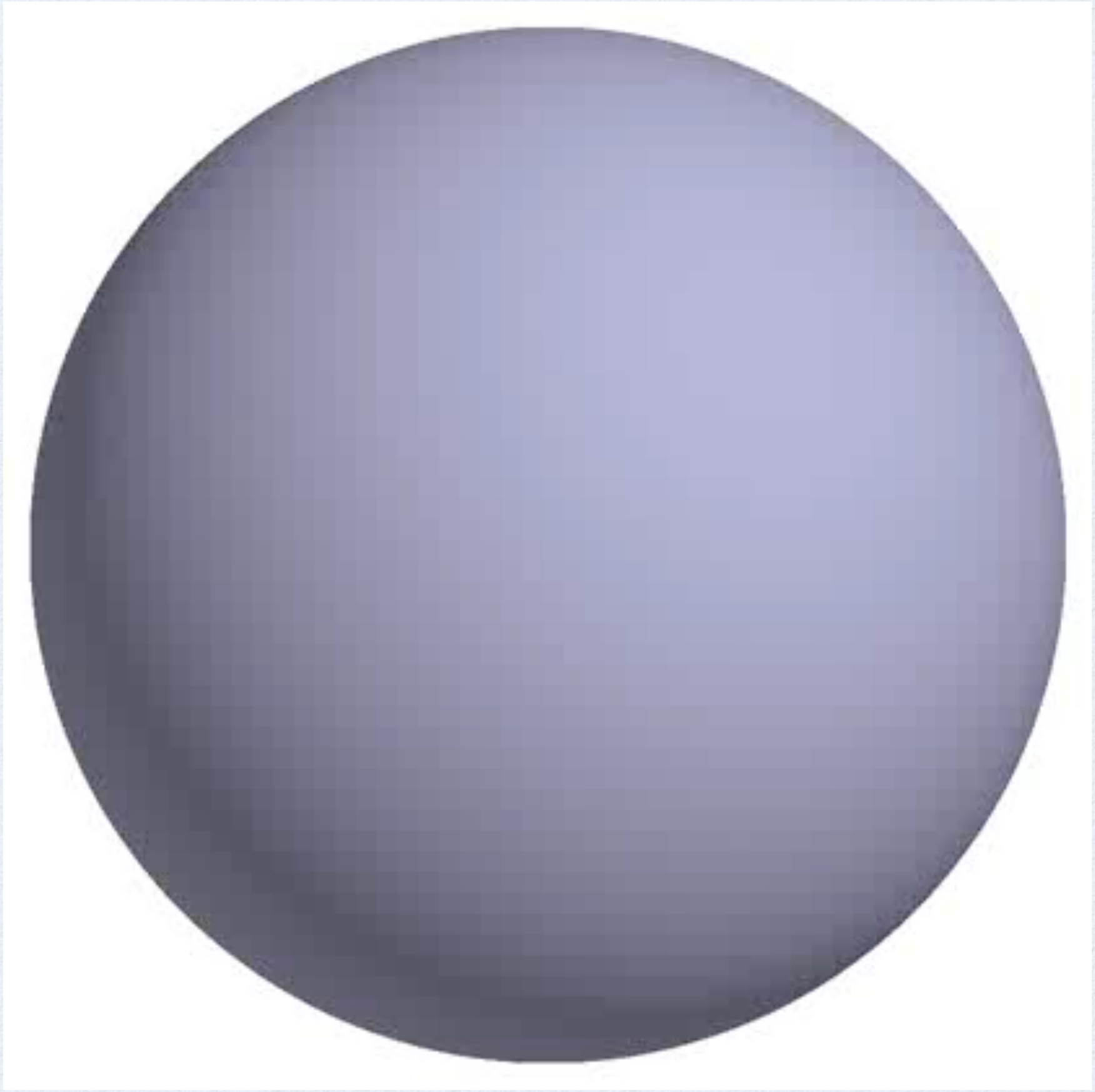


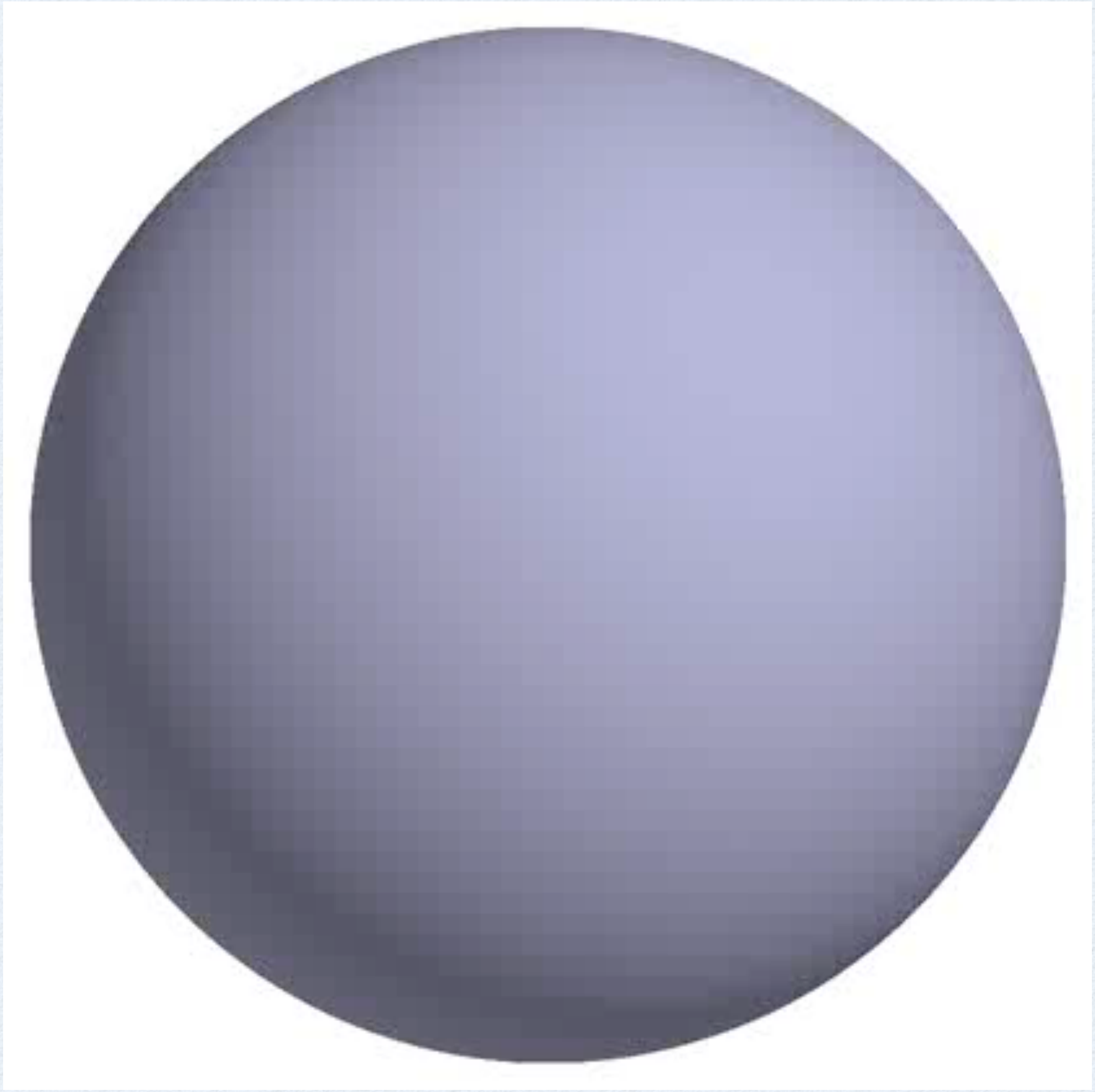
the rabbit

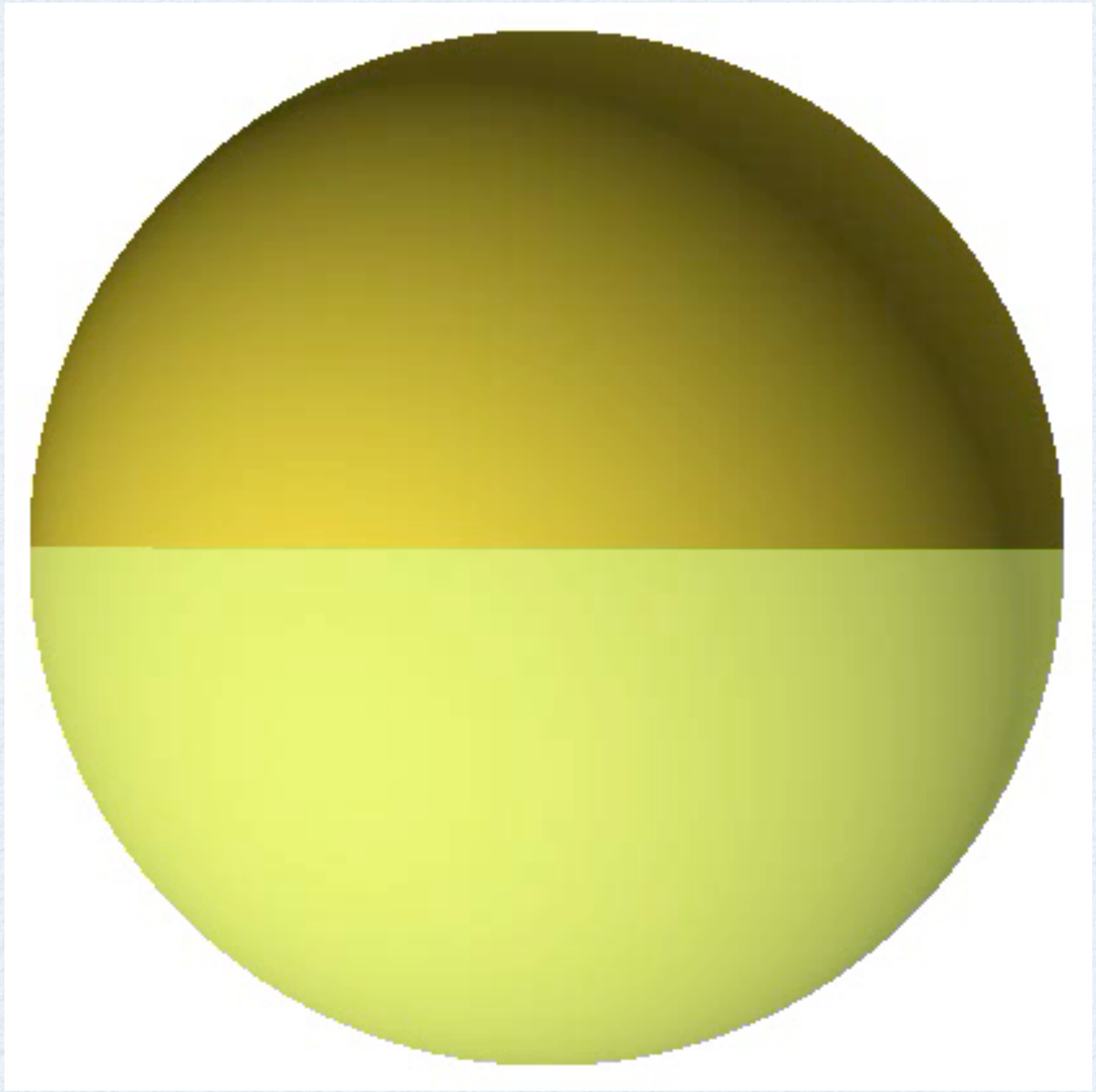
The *mating* of the basilica and the rabbit

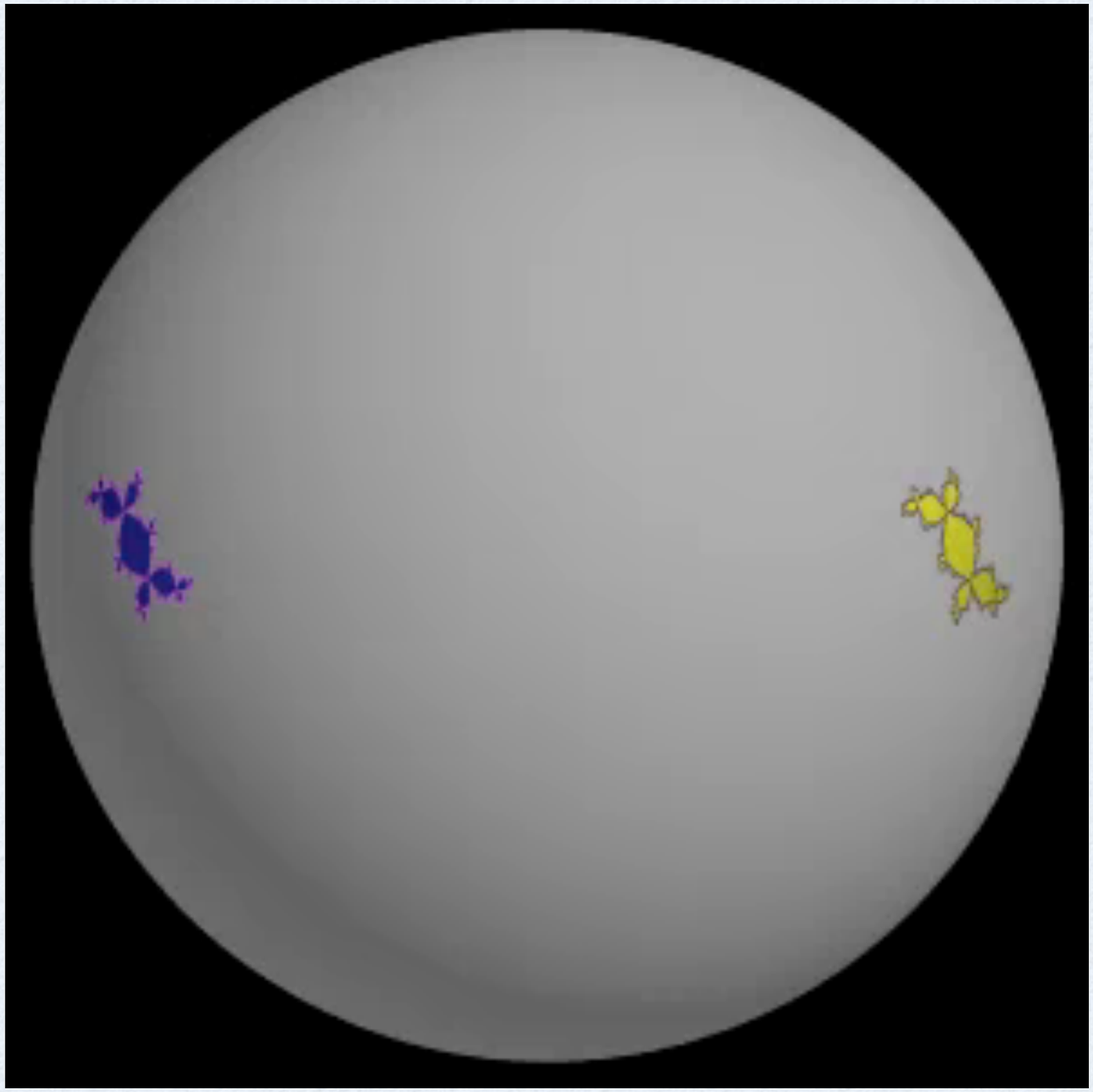


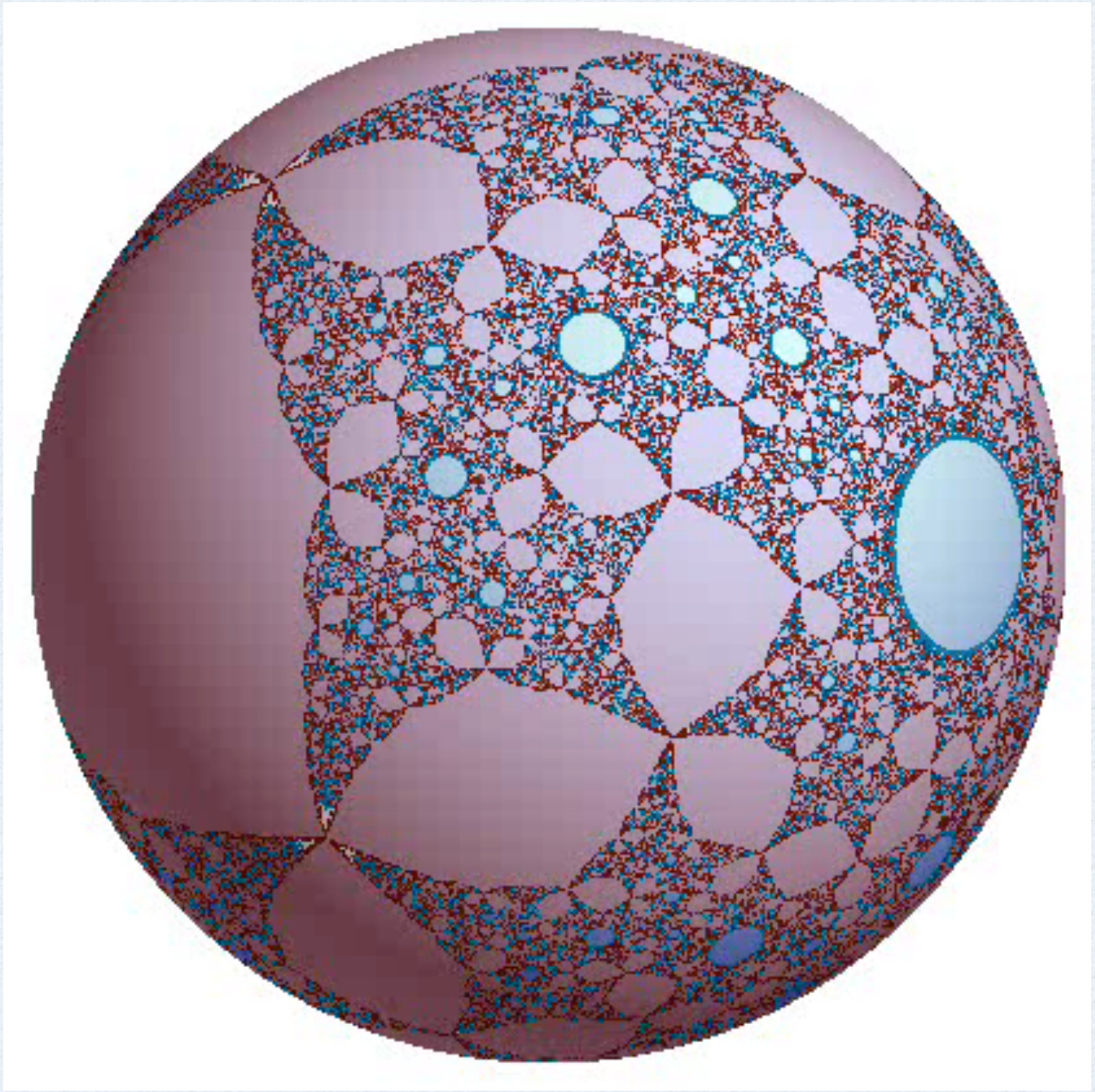
$$F(z) = \frac{2z^2 + 1 - \sqrt{3}}{2z^2 - 2}$$

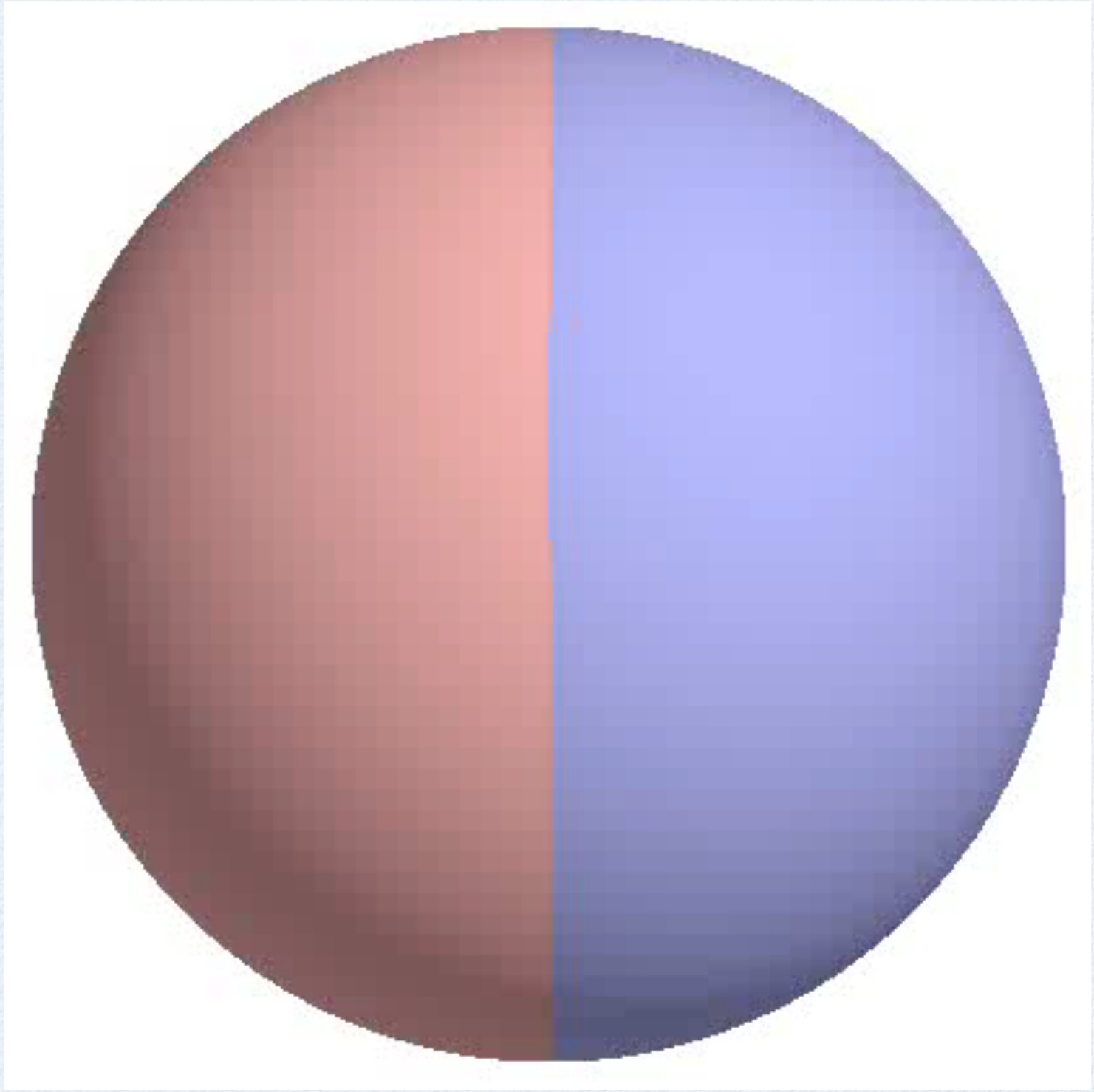


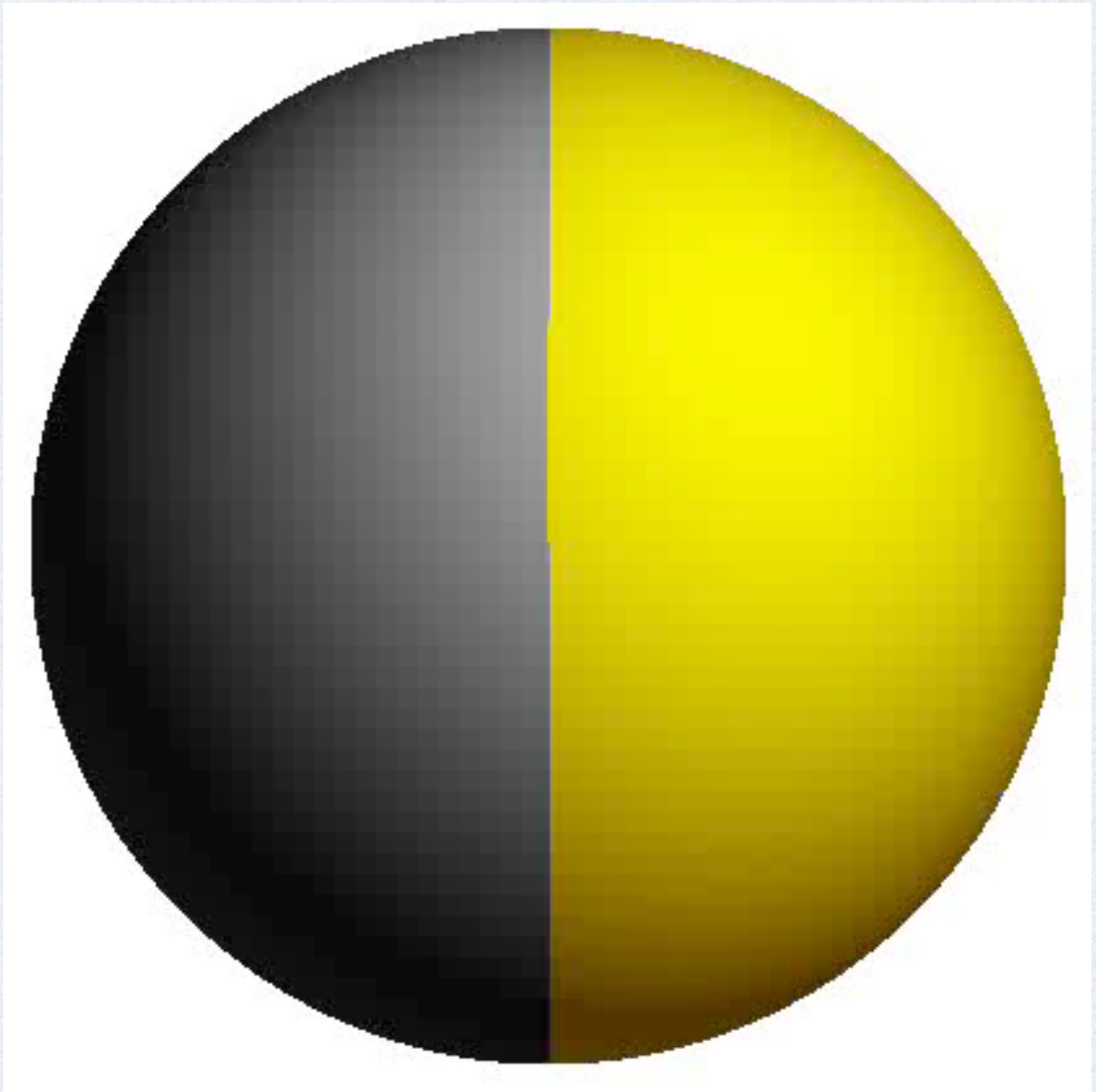


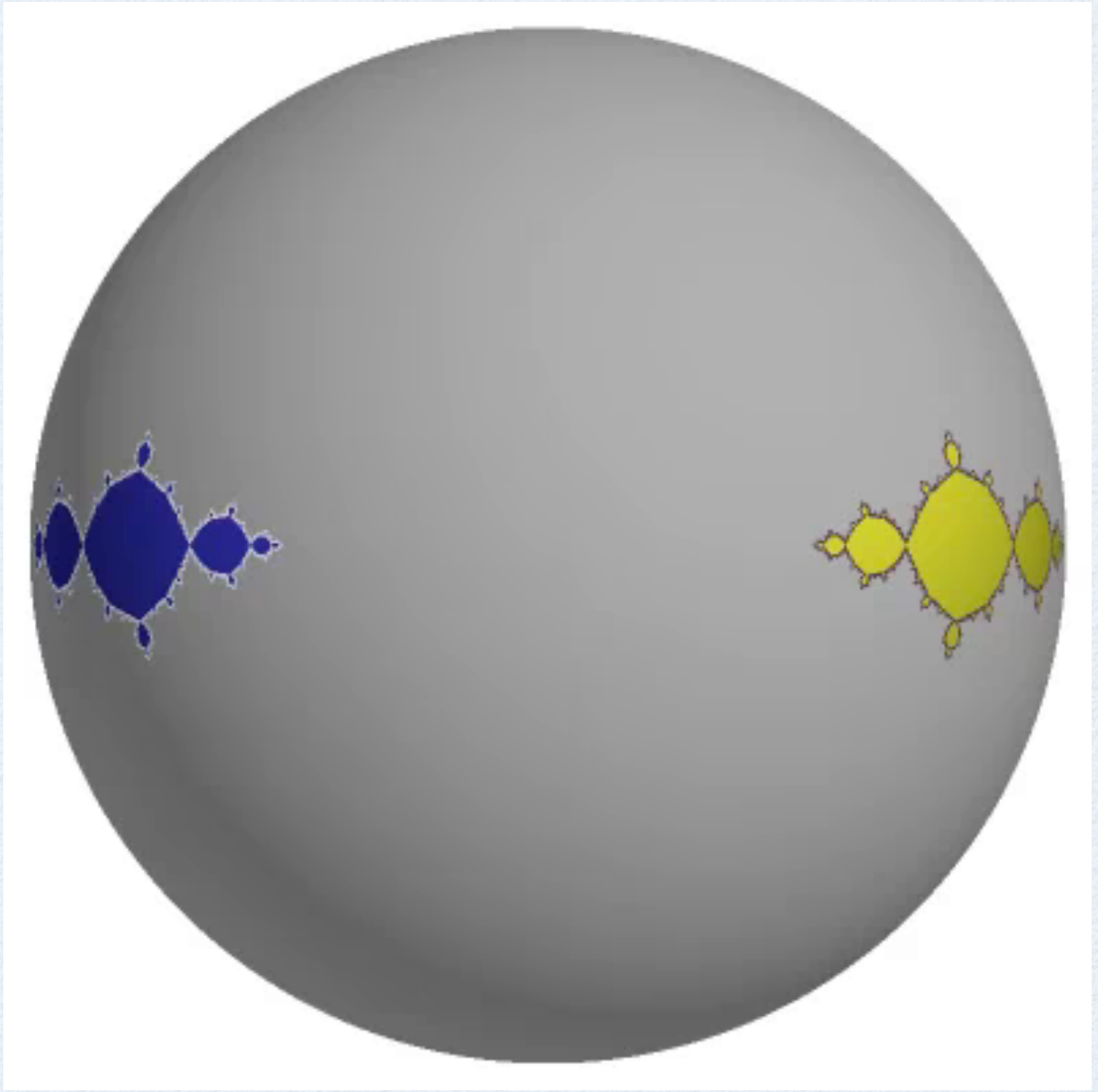




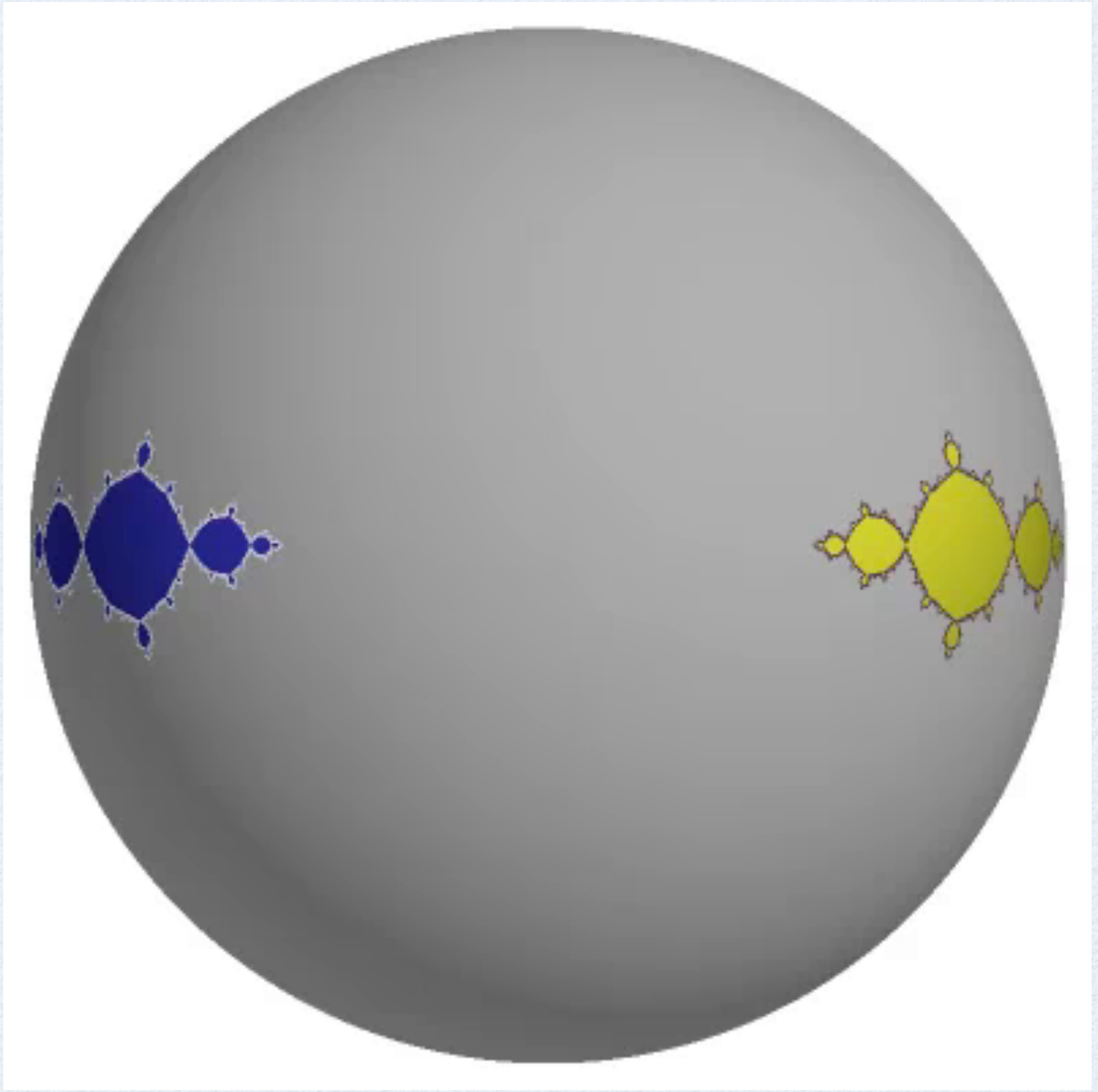


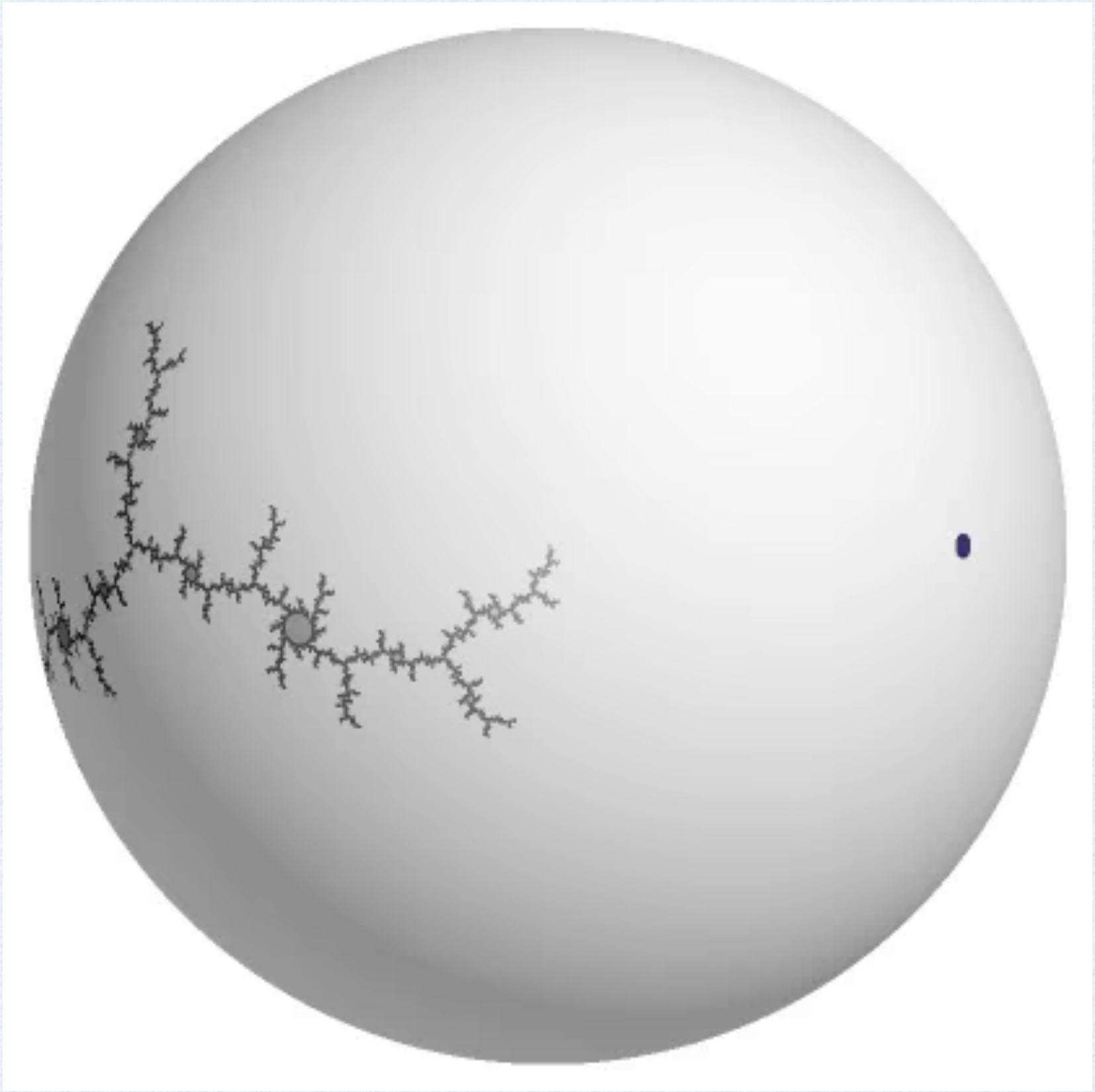




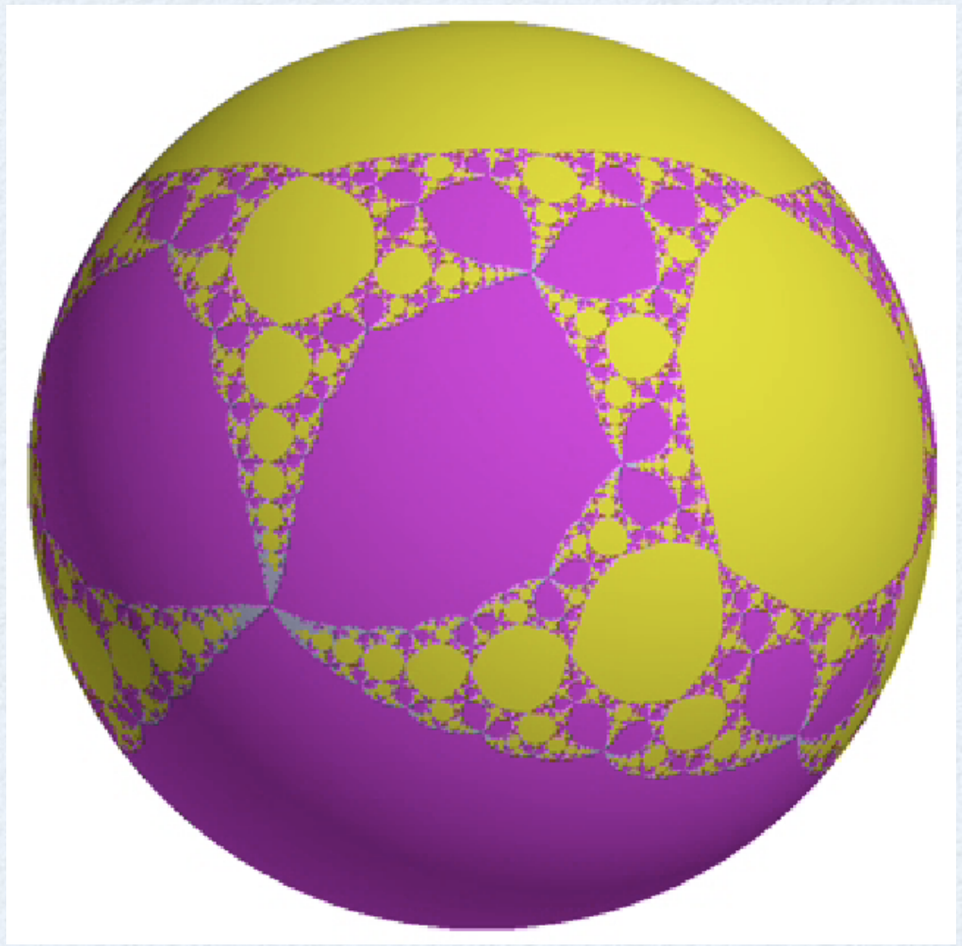


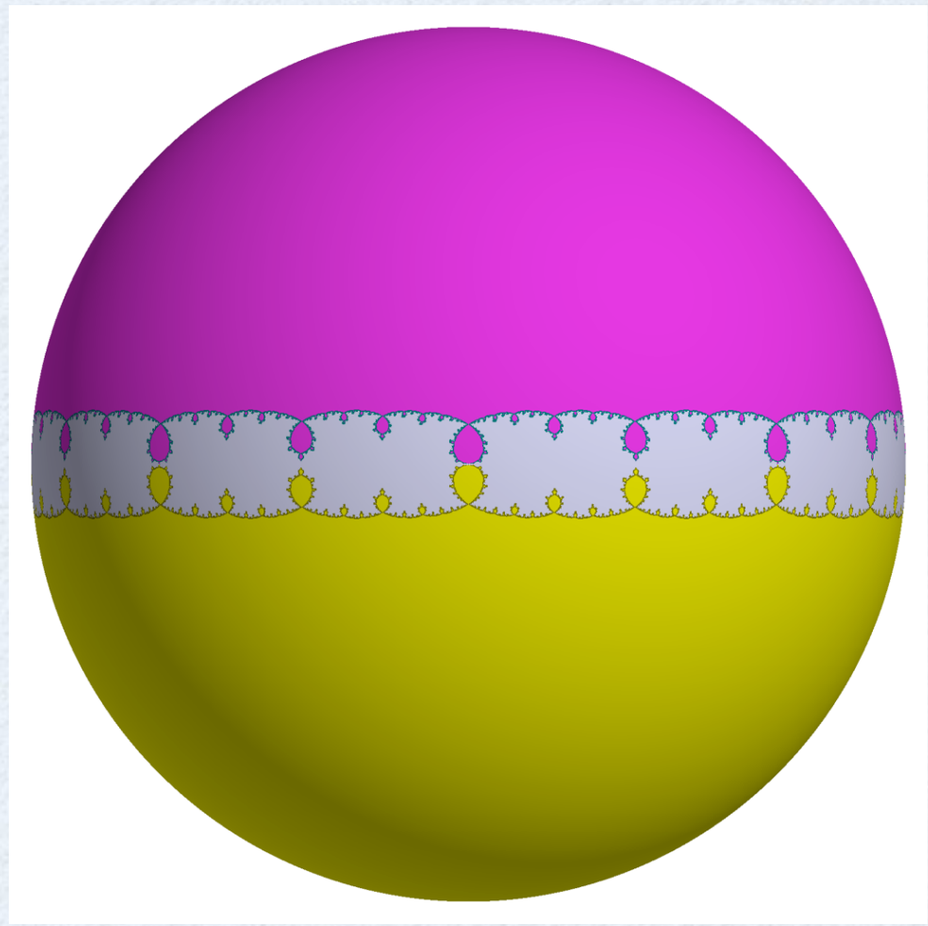
hmmm... let's see that again.

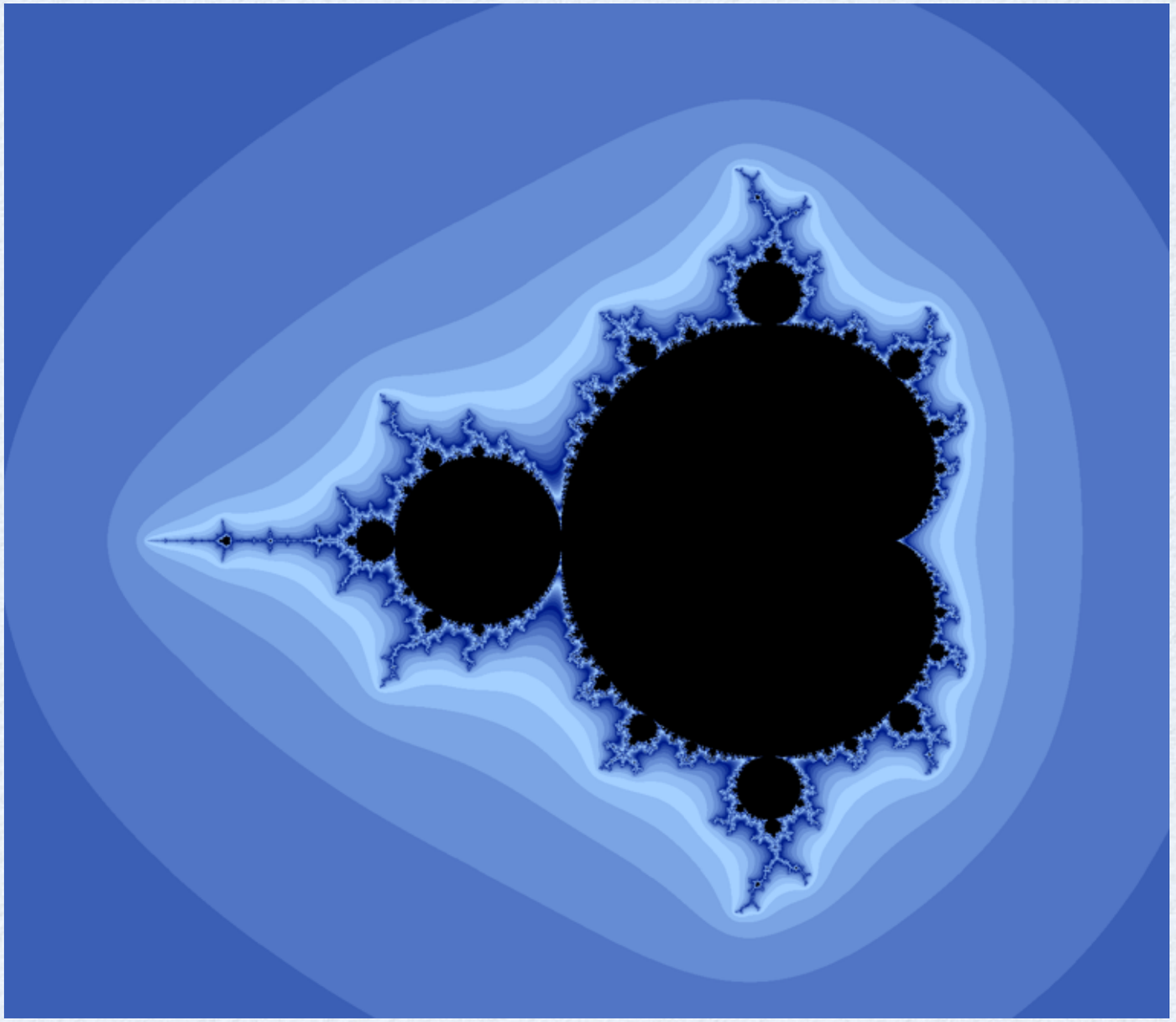




Which quadratic polynomials can be mated?

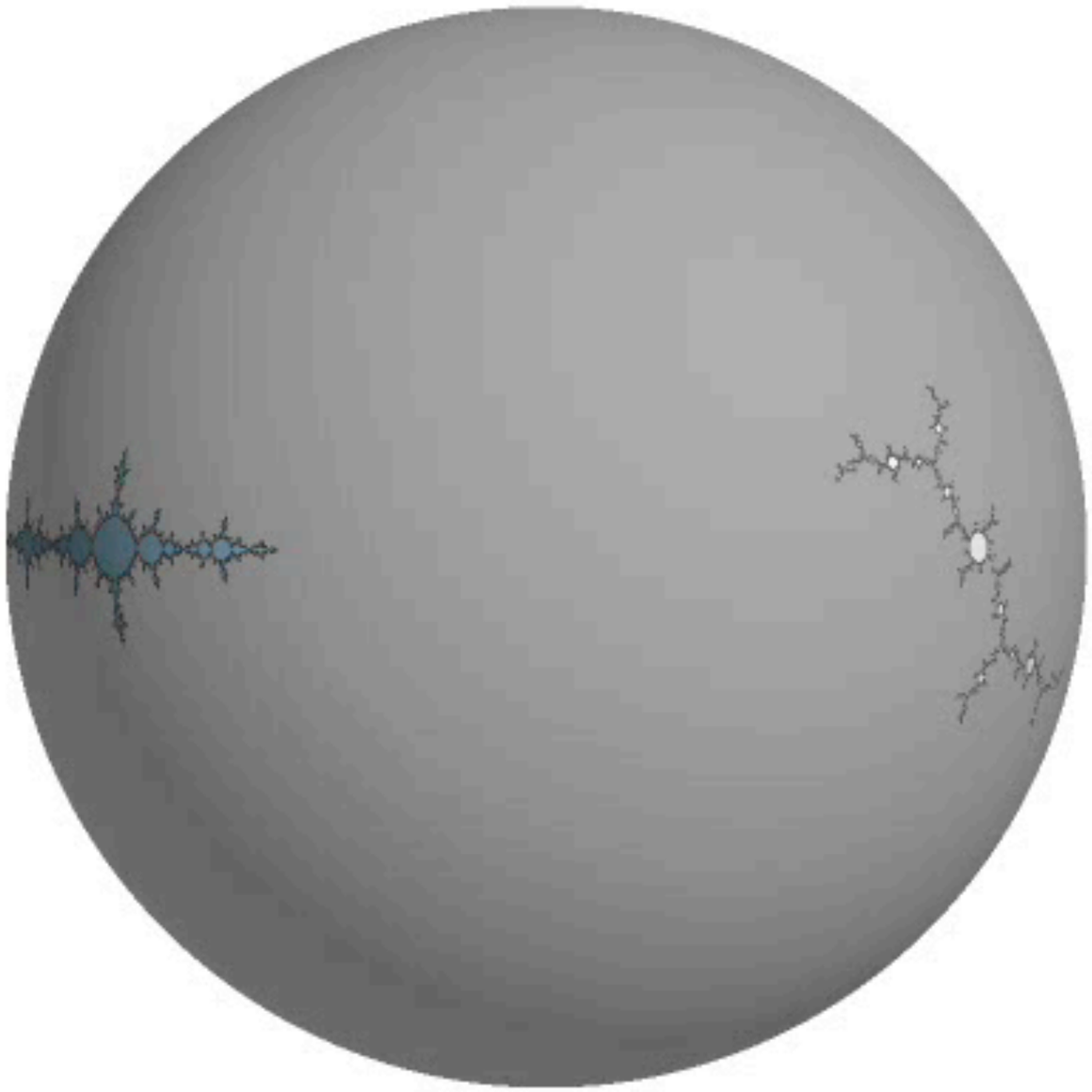


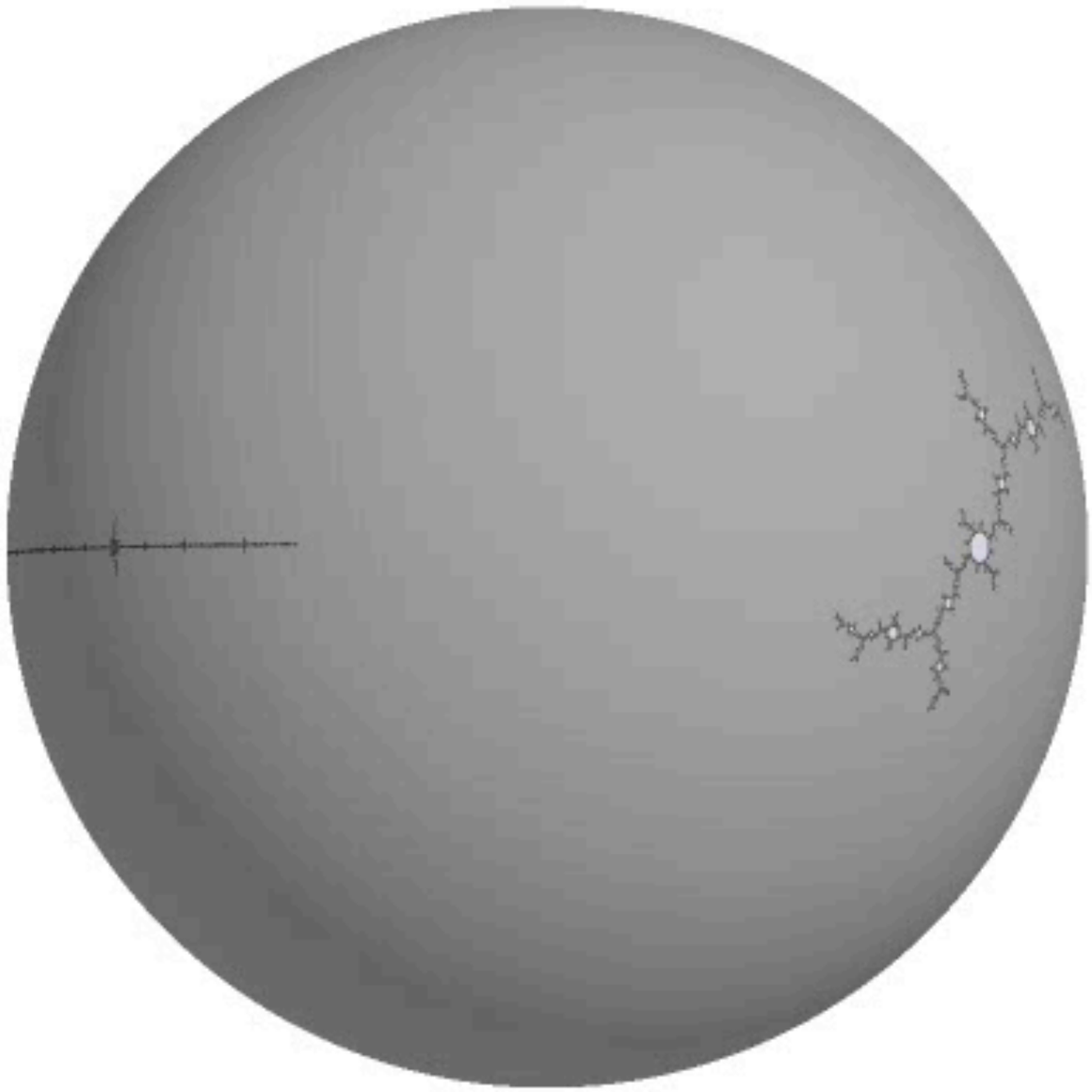




Theorem. (Tan Lei, Rees, Shishikura) Let $p : z \mapsto z^2 + c_1$ and $q : z \mapsto z^2 + c_2$ be postcritically finite. Then p and q can be mated if and only if c_1 and c_2 do not belong to *conjugate limbs* of the Mandelbrot Set.

A shared mating





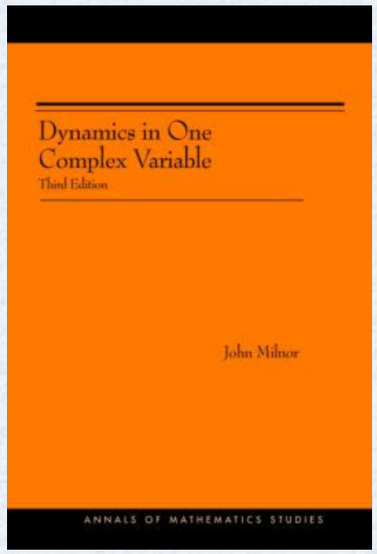
Arnaud Cheritat

polynomial matings:

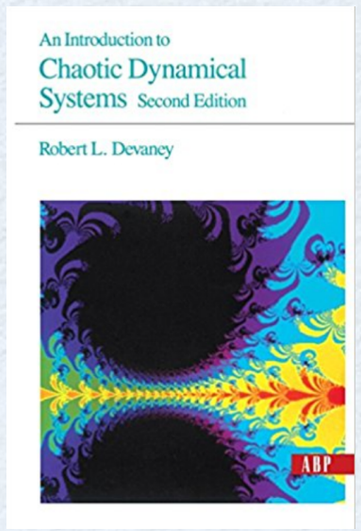
https://www.math.univ-toulouse.fr/~cheritat/MatMovies/

Software: Fractal Stream
Dynamics Explorer
Mandel

Books:

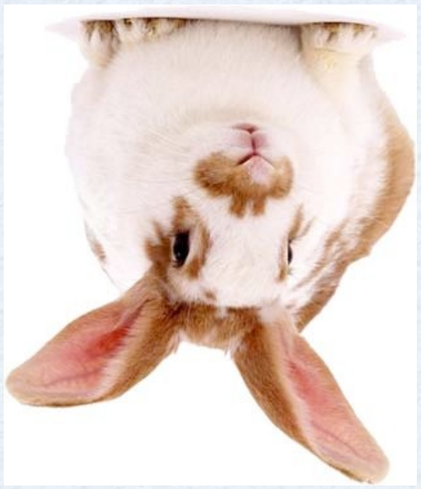


Dynamics in one complex variable
by John Milnor



An introduction to chaotic dynamical systems
by Robert Devaney

Classes: complex analysis, topology,
differential geometry, algebraic topology



Thank you!